Inventory Management with Advance Demand Information and Flexible Delivery

Tong Wang\textsuperscript{1}    Beril Toktay\textsuperscript{2}

\textsuperscript{1}Decision Sciences Area
INSEAD

\textsuperscript{2}College of Management
Georgia Institute of Technology

January 2008
An Example

Point Processes and Queues: Martingale Dynamics (Advances in I Geochemistry) (Hardcover)
by Pierre Brémaud (Author)

Price: £51.50 & this item Delivered FREE in the UK with Super Saver Delivery. See

Availability: Usually dispatched within 7 to 12 days. Dispatched from and sold by Amazon.co.uk.

24 used & new available from £41.62
Traditional Inventory Model

Scarf (1960), with supply leadtime $L$

- Cost structure: ordering, holding, shortage
- Main results:
  - The system can be characterized by the Inventory Position
  - $(s, S)$ policy is optimal
- Immediate delivery
Inventory Model with ADI – Homogeneous Customer Base

Hariharan and Zipkin (1995), discretized version

- Demand information arrives $T$ periods (*demand leadtime*) in advance
- Main results:
  - Symmetry between supply leadtime $L$ and demand leadtime $T$
  - *Effective leadtime* $= L - T$
- **Homogeneous customers**
Gallego and Özer (2001)

- Demand leadtime $T$ is heterogeneous
- Main results:
  - Modified Inventory Position (MIP) = Inventory Position - advance demands
  - Modified state-dependent $(s(V), S(V))$ policy is optimal
- **Exact delivery**: Demand $d_i^j$ has to be satisfied exactly in period $j$
Inventory Model with ADI and Flexible Delivery

Definition

**Flexible Delivery**: Demand $d^i_j$ is allowed to be fulfilled within the time-window $[i, j]$.

Research Questions:

- What is the optimal inventory policy?
- What is the benefit of flexible delivery?
- How does flexible delivery interact with advance demand information?
Inventory Model with ADI and Flexible Delivery

Overview of the Models

I. Homogeneous customers

• FCFS allocation is optimal
• Existing results still hold

II. Heterogeneous customers

• Challenge: Demand Cross-over
• Approximation and heuristics
Traditional Model

Single-period Loss Function ($L = 0$)
Homogeneous ADI Model with Delivery Flexibility

Single-period Loss Function \((L = 0, T = 2)\)

\[
L(X_{i+1}) = x_i + z_i - v_i^i - v_i^{i+1} - d_i^{i+2}
\]

Wang and Toktay

Advance Demand Information and Flexible Delivery
Theorem

The system can be characterized by the

\[ \text{Modified Inventory Position} = \text{Inventory Position} - \text{Advance Demands} \]

and an information state \( \hat{V} \). And a state-dependent \( (s(\hat{V}), S(\hat{V})) \) policy is optimal.
Numerical Illustration
Optimal Cost as a function of $L$ and $T$
Demand Cross-over in the Heterogeneous Model

- Replenishment: $z_i$
- Inventory: $x_i$
- AD Profile Vi: $v_i, v_{i+1}$
- Demand in $i$: $d_i, d_{i+1}, d_{i+2}$
Demand Cross-over in the Heterogeneous Model

Replenishment

Inventory

AD Profile $V_i$

Demand in $i$

$z_i$

$x_i$

$V_i$

$V_{i+1}$

$d_i$

$d_{i+1}$

$d_{i+2}$

Wang and Toktay

Advance Demand Information and Flexible Delivery

12 / 21
Demand Cross-over in the Heterogeneous Model

- Replenishment: $z_i$
- Inventory: $x_i$
- AD Profile $V_i$
- Demand in $i$: $d_i^1$, $d_i^{i+1}$, $d_i^{i+2}$
Demand Cross-over in the Heterogeneous Model

Replenishment

Inventory

AD Profile Vi

Demand in i

Demand in i+1

Wang and Toktay
Advance Demand Information and Flexible Delivery

12 / 21
Approximation

$L = 0$ and $T = 2$

- The allocation assumption (Eppen and Schrage 1981)
- Convex single-period loss function $\Rightarrow$ optimality of $(s(V), S(V))$ policy

Policy is not implementable; cost is a lower bound of the original problem
Protection Level Heuristics

$L = 0$ and $T = 2$

- Reserve protection stocks before satisfying non-urgent demands

- Three protection levels considered:
  - PL($0$)–zero protection: $\sigma_i = 0$
  - PL($\Sigma$)–full protection: $\sigma_i$ covers the whole support of $d_{i+1}^{i+1}$
  - PL($\sigma$)–“optimal” protection:
    \[
    \sigma_i = \arg \min \left\{ h \cdot \sigma_i + p \cdot \mathbb{E}\left[(d_{i+1}^{i+1} - \sigma_i)^+\right] \right\}.
    \]
Protection Level Heuristics

$L = 0$ and $T = 2$

Quasiconvex Single-period Loss Function
Numerical Comparison

A Typical Case \((L = 0, T = 2; K = 100, h = 1, p = 9)\)
Numerical Comparison

An Extensive Test \((L = 0, 1, 2, 3, 4; K = 50, 100, 200; h = 1, 3, 5; p = 9, 19, 29)\)

**Optimality gap**

![Graph showing Optimality gap with different values of L, K, h, and p]

**Cost saving due to delivery flexibility**

![Graph showing Cost savings with different values of L, K, h, and p]

(Full height — AP vs. ADI, darker bar — PL(σ) vs. ADI)
Conclusion

• Contributions
  • Optimality of the State-dependent \((s(V), S(V))\) policy in the homogeneous model
  • Implementable and near-optimal (2% optimality gap) heuristics for the heterogeneous case

• Managerial Implications
  • Significant cost savings from delivery flexibility: 14%
  • Asymmetry: extending demand leadtime is more beneficial than shrinking supply leadtime
  • Advance demand information and delivery flexibility are complements
  • Win-win solution for both parties

• Future Research
  • Pricing and incentive scheme design
  • Consumers’ strategic behavior in response to flexible delivery under repeated interactions