Newsvendor Pricing Problem in a Two-Sided Market

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Abstract

We study the pricing problem of a “platform” intermediary who needs to jointly determine the selling price of the platforms (hardware) sold to consumers and the royalty charged to content developers for content (software). We assume that the market is two-sided, that is, that the demands for content and for platforms are interdependent. This problem is a generalization of the classic newsvendor pricing problem, and is motivated by the recent surge of interest in the study of two-sided markets. The former focuses mainly on the impact of pricing on demand and the optimal replenishment strategy, whereas the latter focuses on the cross-side network effect of the two markets faced by the platform intermediary. Integrating the two areas of research, our model elucidates the impact of supply chain replenishment costs and demand uncertainty on the strategic issues of platform pricing in a two-sided market.

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1 Introduction

In the classic newsvendor pricing model, the challenge is to determine jointly the price and the ordering quantity of a product when demand for the product is a (random) function of the price charged (cf. Petruzzi and Dada [10] and the references therein). Raz and Porteus [11] motivated the use of conjoint data in this model with the following example: “... when a company launches a new product, such as software giant Microsoft launching its Xbox 360 at the end of 2005 (Business Week 2005), it must estimate the relationship between the price it sets and the demand distribution that it will face.” Contrary to earlier assumptions used in the literature, they argued that demand variability may not be monotonic in the pricing decision (e.g., some products may have lower demand variability at very high or very low prices); hence, the optimal pricing solution for the newsvendor problem may not even be monotone in the wholesale price of the product. It is thus important to model accurately the demand-price relationship in the newsvendor pricing problem.

Unfortunately, for products like the Xbox 360, the demand-price relationship may be so complex that it cannot be captured by conjoint data collected at the consumer end alone. Each consumer needs an Xbox 360 console to play the console games developed by the content developers. These game developers can be divided into two types - “first party” and “third party.” First-party games are developed in-house, while third-party games are developed by external parties. Microsoft profits from the software market in two ways: (i) royalties charged to the sales of each copy of the game title, and (ii) margins from the sales of software in the case of first-party games. The amount of royalties charged affects the developmental efforts of the games, which in turns affects the quality, and thus the sales, of the software. Interestingly, Microsoft’s business strategy for Xbox is to gain consumer market share by selling each console at a subsidized rate. Cutting prices on consoles make them more attractive to consumers, who in turn buy the games, which in turn generates additional profits. One popular title that has generated a lot of revenue is the Halo 3 game played on the Xbox 360, for which Microsoft has exclusive rights.\(^1\)

\(^1\)As of 2008, two years since product launch, about 20 million Xbox 360 consoles have been sold. Faced with competition for consumers from competitors like Sony and Nintendo, it is critical that Microsoft gain as much of the console market share as possible. Microsoft’s strategies for profitability in the Xbox business are (i) driving costs down, (ii) increasing game sales, (iii) selling more accessories (like controllers, batteries, etc.), and (iv) increasing memberships to its Live Marketplace. The above information is based on personal communication.
In the Xbox supply chain, Microsoft plays more the role of a “platform” intermediary. In this market, there is a strong cross-side network effect, that is, the value of the product on one side of the platform intermediary is correlated to the number of users on the other side. This property is also known as an indirect network externality. Such network externalities play an important role in the pricing strategies of intermediaries in many two-sided markets, and provide the theoretical basis to justify the pricing strategy adopted by numerous two-sided markets - where one side of the market is often treated as a profit center, while the other is treated as a loss leader, or at best financially neutral.

In this paper, we examine a class of pricing and inventory problems faced by such platform intermediaries in a two-sided market. We need to determine jointly the price of each unit of the platform, and the royalty charged to the software developers for each unit of software sold. We propose a model to integrate the effects of indirect network externalities into the newsvendor pricing decisions. Recent results in the two-sided network literature synthesizes the spillover effect with optimal pricing decisions (cf. Parker and van Alstyne [9]) and shows that the optimal pricing decision necessitates subsidizing one side of the market so that profits can be accrued at the other side. Interestingly, despite adding supply chain operational costs in our model, such peculiar pricing behavior in the optimal solution may persist, though we discover another strategic option: the intermediary could also charge a surplus to both sides of the market to compensate for supply chain costs, despite the positive indirect network externalities in the markets. This appears to be the preferred strategy in the high-end fashion magazine industry, where both readers and advertisers are charged a surplus despite the positive network externality of readership on the advertising market. The key contribution of this paper is its complete characterization of the different regimes of strategic pricing options, demonstrating clearly the importance of supply chain operational concerns on the strategic pricing decisions of a firm operating in a two-sided market.

2 Literature Review

Interest in understanding two-sided markets is relatively new (cf. Rochet and Tirole [12, 13] and the references therein). Their recent popularity is mainly the result of the need to explain the workings of the software market and related industries. One of the most counter-intuitive observations in a two-sided network market is that profits can still accrue to the intermediary
even if one side of the market is heavily subsidized by the intermediary. This is the case for firms like Adobe, which basically distribute their products free to one side of the market: “...consumers of a portable document reader may, at their discretion, forgo the cost of the portable document creator now and forever” [9].

Parker and van Alstyne [9] study this phenomenon with the key goals of providing the conditions that determine the optimal pricing strategy for the intermediary and determining which side of the market to subsidize. They assume that the demand on both sides is deterministic with respect to the prices, thus preclude analysis of the impact of product shortages on the performance of the pricing strategy. Such concerns are, however, pertinent in two-sided markets such as the Xbox supply chain, where the long supply/production lead time and short selling season means that the intermediary has to acquire enough consoles based on forecasts long before the selling season begins. Demand uncertainty and supply shortages are major concerns in this market. This issue is also pertinent in the newspaper industry, where readers and advertisers represent two sides of the market, although unlike the Xbox market, the large number of advertisements in the newspaper may negatively affect the newspaper’s circulation figures. Hence, although the number of readers has a positive “spillover” effect on the advertisers, the number of advertisers may have a negative spillover effect on the number of readers.

Works on two-sided markets include Argentesi and Filistrucchi [1], Armstrong [2], Caillaud and Jullien [3], Parker and van Alstyne [9], and Rochet and Tirole [12, 13]. Rochet and Tirole [12] discuss the concept of “multi-homing” on one side of the market. Multi-homing arises when different platforms compete for a bigger share of one side of a two-sided market. The authors introduce a single homing index on the other side of the market to measure the effectiveness of the platform in “steering” multi-homing users from not doing so. Caillaud and Jullien [3] also consider a two-sided market with competition among intermediaries. Complementing Rochet and Tirole [12], they also look at the concept of multihoming. They find that “due to indirect network effects, the key pricing strategies are of a “divide-and-conquer” nature, subsidizing the participation of one side (divide) and recovering the loss on the other side (conquer).” Parker and van Alstyne [9] give the conditions under which a company should subsidize. Argentesi and Filistrucchi [1] present a case study on the Italian newspaper industry. Armstrong [2] gives a useful survey of the results in Parker and van Alstyne [9] and others as to which side
of a market should be subsidized and whether the outcome will be socially efficient.

A two-sided network externality differs from the classic network externality (cf. Katz and Shapiro [8], Farrell and Saloner [6]), the latter being an intramarket network externality while the former is an intermarket network externality. Also, our consideration of subsidizing one market has precedence in multi-product pricing. However, a two-sided network externality differs from traditional tied sales literature (cf. DeGraba [4], Whinston [14]), in that one market need not consume the complementary good. For example, consumers of a free Adobe reader may, at their discretion, forgo the cost of the portable document creator now and forever.

There is by now a huge literature on the classic newsvendor model. Raz and Porteus [11] (see also the references therein), to the best of our knowledge, is the first to analyze this problem using more realistic conjoint data. The demand function, as a response to price, is random and is calibrated from conjoint experiments. It thus contains the additive and multiplicative demand models as special cases. Unlike our work, Raz and Porteus [11] do not take into account how the cross-network effects of one side of the market affect the demand function on the other side, as they focus solely on one side of the market.

3 Demand Models

In the following model, we take the case of the Xbox as the underlying model. This model can also be applied to other appropriate situations. The platform intermediary serves as a link for the cross-network effect between the content provider side and the end consumer side, where demand from one side affects the demand from the other, and vice versa.

The key decision variables in our model are as follows:

- \( p_c =: \) price charged to consumers for an Xbox console.
- \( p_j =: \) price (royalty) charged to content providers for each unit of software sold.

Let \( q_c(p_c, p_j, Y) \) and \( q_j(p_c, p_j, Y) \) denote the demand for consoles and the total units of software sold in the market, respectively. Our demand model assumes that

\[
q_c(p_c, p_j, Y) = D_c(p_c, Y) + e_{jc} \times D_j(p_j), \quad (1)
\]

\[
q_j(p_c, p_j, Y) = D_j(p_j) + e_{cj} \times D_c(p_c, Y). \quad (2)
\]
Here, \( D_c \) denotes the (inherent) demand from consumers for Xbox consoles without taking into account the cross-network effect. It depends on \( p_c \) and a random parameter \( Y \), which has a positive probability density function \( f_Y(y) \) and cumulative density function \( F_Y(y), y \in \mathbb{R} \).

The parameter \( Y \) denotes the market elements that affect console demand but that cannot be attributed to the pricing decision. We assume \( E_Y(Y) = 0 \). Similarly, \( D_j \) denotes the expected (inherent) demand for software products without taking into account the cross-network effects. This is a function of \( p_j \), since the royalties charged to the content providers affect their willingness to develop new software titles, and thus indirectly affect the number of software titles developed for the platform which subsequently affect the level of \( D_j \).

We augment the demand models with the additional terms of \( e_{jc} D_j(p_j) \) and \( e_{cj} D_c(p_c, Y) \) in (1) and (2), respectively, to capture the indirect network effects in the two-sided market. With a positive \( e_{jc} \) and \( e_{cj} \), the sales on one side of the market can thus grow by increasing the sales on the other side. This demand model ((1) and (2)) is motivated by the work in Parker and van Alstyne [9]. A slightly different way to model the internetwork effect (essentially a multiplicative formulation) can be found in Rochet and Tirole [12]. Note that there is a subtle element in our model - while the spillover effect from the console market to the software market depends on actual console sales (and is thus random), the spillover effect from the software market to the console market depends on the average software sales. This distinction is natural, as each sale of a console is normally accompanied by sales in software, whereas the reverse may not be true. Furthermore, the purpose of this paper is to understand the effect of supply chain costs on the pricing decisions in a two-sided market and the above formulation allows us to isolate the impact of demand uncertainty in the console market on the optimal pricing strategy.

Upon manipulating (1) and (2), we obtain

\[
q_c(p_c, p_j, Y) = (1 - e_{cj} e_{jc}) D_c(p_c, Y) + e_{jc} q_j(p_c, p_j, Y), \tag{3}
\]

\[
q_j(p_c, p_j, Y) = (1 - e_{cj} e_{jc}) D_j(p_j) + e_{cj} q_c(p_c, p_j, Y). \tag{4}
\]

Let us consider (3). Differentiating \( q_c \) w.r.t. \( p_j \) using (3), we obtain \( \frac{\partial q_c}{\partial p_j} = e_{jc} \frac{\partial q_j}{\partial p_j} \), while differentiating \( q_j \) w.r.t. \( p_c \) using (4), we obtain \( \frac{\partial q_j}{\partial p_c} = e_{cj} \frac{\partial q_c}{\partial p_c} \). The parameters \( e_{jc} \) and \( e_{cj} \) have the following natural interpretations:

- \( e_{jc} \) denotes the content-to-console internetwork externality term, which measures how
much purchases on the content side affect sales of the console market, while

• \( e_{cj} \) denotes the console-to-content internetwork externality term, which measures how much purchases on the console side affect the purchases of copies of game titles in the content market.

Let

\[
\overline{D}_c(p_c) = E_Y[D_c(p_c, Y)];
\]

\[
\overline{q}_c(p_c, p_j) = E_Y[q_c(p_c, p_j, Y)];
\]

and

\[
\overline{q}_j(p_c, p_j) = E_Y[q_j(p_c, p_j, Y)].
\]

We define

• \( \frac{\partial \overline{q}_c}{\partial p_j} \) = a two-sided network externality, or spillover effect, from the content-provider side to the consumer side;

• \( \frac{\partial \overline{q}_j}{\partial p_c} \) = a two-sided network externality, or spillover effect, from the consumer side to the content provider side; and

• \( r = \frac{\partial \overline{q}_c}{\partial p_j} = \frac{\partial \overline{q}_j}{\partial p_c} \) = ratio of spillover effects.

The parameter \( r \) plays a key role in the two-sided network literature, as the optimal pricing behavior (which side to subsidize) can be connected to this single parameter in the existing theory.

To wrap up our discussion of the demand models, we further assume that

\[
D_c(p_c, Y) = \overline{D}_c(p_c) + H_c(p_c)Y,
\]

where either \( H_c(p_c) > 0 \) for all \( p_c \), or \( H_c(p_c) = 0 \) for all \( p_c \). The latter reduces to the classic two-sided market problem, since there is no demand uncertainty. Henceforth, we assume the more interesting case, where \( H_c(p_c) > 0 \) for all \( p_c \). This form of the demand function also appears in [11] and includes additive (when \( H_c(p_c) = \) constant) and multiplicative (when \( H_c(p_c) = \overline{D}_c(p_c) \)) demands as special cases. More importantly, as \( H_c(p_c) \) need not be monotonic in \( p_c \), the above
demand model allows us to handle the situation where the variability of demand could be small for either low or high values of $p_c$. This appears to be the norm, rather than the exception, in most settings.

3.1 Handling demand uncertainty

Demand uncertainty in the console markets has different repercussions from that in the software markets. Because of the long lead time in the procurement and production cycle, as well as the short selling season, the number of consoles to produce is normally decided based on demand forecast. This creates a situation where the supply and demand for consoles may be mismatched, leading to loss of efficiency in the supply chain of the console market. On the other hand, we assume that the software titles can be produced based on real-time demand data, and hence ignore the occasions where certain software titles may be sold out.

To cope with demand uncertainty, let

- $x$ denote the total number of the commodity (e.g., Xbox consoles) that will be produced at the beginning of the selling season.

The decision for $x$ is affected by the following parameters:

- $w$: wholesale price per unit of commodity.
- $s$: salvage value per unit of commodity.
- $p$: penalty cost per unit of commodity shortage.

Note that $w > s$. When the supply component of the platform of interest is brought into the picture, the total profit that can be attained is given by

$$
\pi(x, p_c, p_j, Y) = p_c \min \{q_c(p_c, p_j, Y), x\} + p_j q_j(p_c, p_j, Y) - wx - p(q_c(p_c, p_j, Y) - x)^+ + s(x - q_c(p_c, p_j, Y))^+.
$$

We would like to choose $x$, $p_c$, and $p_j$ to maximize the expected total supply chain profit $E[\pi(x, p_c, p_j, Y)]$. In the rest of the paper, for ease of exposition, we assume the following:
Assumptions

A. $D_c$ is nonnegative, and decreases as $p_c$ increases. $D_j$ is nonnegative, and decreases as $p_j$ increases.

B. $e_{cj}e_{jc} < 1$, omitting the degenerate case when $e_{cj}e_{jc} = 1$. Also, we assume that $e_{cj} \geq 0$, which means that sales of Xbox consoles always have a nonnegative internetwork effect on sales of titles.

C. Any optimal solution $(x^*, p^*_c, p^*_j)$ to (6), where $\pi(x, p_c, p_j, Y)$ is given by (5), is such that $x^* > 0$, $\overline{q}^*_c > 0$, and $\overline{q}^*_j > 0$.

D. When $e_{cj} = e_{jc} = 0$, $(p^*_c, p^*_j, y^*_0)$ is the optimal solution to (9) if and only if it satisfies the KKT conditions for (9). Also, it is the only optimal solution to (9).

Assumption A is a standard assumption used to describe how a demand function responds to changes in price. Assumption B is an assumption used in the literature on two-sided network effects, as in Parker and van Alstyne [9]. Assumptions C and D are technical assumptions introduced for ease of exposition, and remove the need to digress to the issues of the existence and uniqueness of the optimal pricing strategy.

4 Optimality Conditions

We would like to study the following maximization problem:

$$\max \quad E_Y(\pi(x, p_c, p_j, Y))$$
subject to $x \geq 0$. \hspace{1cm} (6)

Let $(x^*, p^*_c, p^*_j)$ be an optimal solution to the maximization problem (6). There exists $y^*_0 \in \mathbb{R}$ such that

$$\overline{D}_c(p^*_c) + e_{jc}D_j(p^*_j) + H_c(p^*_c)y^*_0 = x^*.$$

Intuitively, we can think of $y^*_0$ as a scaled difference between actual expected consumer demand and the supply of Xbox consoles at optimality. Using this observation, we can rewrite (6) as

$$\max \quad E_Y(\pi(x, p_c, p_j, Y))$$
subject to $x \geq 0$

$$\overline{q}_c(p_c, p_j) + H_c(p_c)y_0 = x,$$ \hspace{1cm} (7)
where
\[ \overline{q}_c(p_c, p_j) = E_Y(q_c(p_c, p_j, Y) = D_c(p_c) + e_{jc} D_j(p_j). \]  
(8)

(7) is equivalent to
\[ \min \quad -E_Y(\pi(\overline{q}_c(p_c, p_j) + H_c(p_c) y_0, p_c, p_j, Y)) \]
subject to \( \overline{q}_c(p_c, p_j) + H_c(p_c) y_0 \geq 0. \)

Let
\[ \overline{q}_j(p_c, p_j) = E_Y(q_j(p_c, p_j, Y)) = D_j(p_j) + e_{cj} D_c(p_c). \]  
(10)

Note that both \( \overline{q}_c \) and \( \overline{q}_j \) depend on \( p_c \) and \( p_j \). Henceforth, we suppress the explicit dependence for ease of exposition. We also use \( H'_c, \overline{q}_c^* \) and \( \overline{q}_j^* \) to denote \( H_c(p_c^*), \overline{q}_c(p_c^*, p_j^*) \) and \( \overline{q}_j(p_c^*, p_j^*) \), respectively.

It can be shown that the objective function can be re-written as
\[ E_Y(\pi(\overline{q}_c + H_c(p_c) y_0, p_c, p_j, Y)) \]
\[ = (p_c - w) \overline{q}_c + p_j \overline{q}_j + (p_j + p - w) H_c(p_c) y_0 + (p_c + p - s) H_c(p_c) \int_{-\infty}^{y_0} (y - y_0)f_Y(y)dy. \]

supply chain cost

Let \( (p_c^*, p_j^*, y_0^*) \) be an optimal solution of (9) associated with that of (6). Then we have \( \overline{q}_c^* + H'_c y_0^* > 0 \), by Assumption C, where we assume that \( x^* > 0 \). From the KKT conditions, we have

**Proposition 1** Let \( (p_c^*, p_j^*, y_0^*) \) be an optimal solution to (9). It then satisfies
\[ p_c^* = \frac{w - s \int_{y_0^*}^{y_0} f_Y(y)dy}{\int_{y_0^*}^{y_0} f_Y(y)dy} - p, \]  
(11)
\[ \overline{q}_c^* = (w - p_c^*) \left( \frac{\partial \overline{q}_c}{\partial p_c} \right)^* - p_j^* \left( \frac{\partial \overline{q}_j}{\partial p_c} \right)^* + H'_c \int_{y_0^*}^{y_0} (y - y_0^*) f_Y(y)dy \]
\[ +(H'_c)^* \left( (w - p_c^* - p) y_0^* + (s - p_c^* - p) \int_{-\infty}^{y_0^*} (y - y_0^*) f_Y(y)dy \right), \]  
(12)
\[ \overline{q}_j^* = (w - p_c^*) \left( \frac{\partial \overline{q}_c}{\partial p_j} \right)^* - p_j^* \left( \frac{\partial \overline{q}_j}{\partial p_j} \right)^* \]  
(13)

**Proof**: It follows from the KKT conditions of (9) that
\[ \begin{pmatrix} -\overline{q}_c - H_c y_0 - H_c \int_{-\infty}^{y_0^*} (y - y_0) f_Y(y)dy + (w - p_c) \frac{\partial \overline{q}_c}{\partial p_c} - p_j \frac{\partial \overline{q}_j}{\partial p_c} + (w - p_c - p) y_0 H'_c \\ + (s - p_c - p) H'_c \int_{-\infty}^{y_0^*} (y - y_0) f_Y(y)dy \\ -\overline{q}_j + (w - p_c) \frac{\partial \overline{q}_c}{\partial p_j} - p_j \frac{\partial \overline{q}_j}{\partial p_j} \\ (w - p_c - p) H_c - (s - p_c - p) H_c \int_{-\infty}^{y_0^*} f_Y(y)dy \end{pmatrix} = 0. \]  
(14)
Therefore, using (14), (11)-(13) follow.

Note that (11) is equivalent to

\[
\text{Prob}(q_c(p_c^*, p_j^*, Y) \leq x^*) = \frac{p_c^* + p - w}{p_c^* + p - s}.
\]  

This is merely the optimality condition for the newsvendor problem with \( p_c^* \) and \( p_j^* \) fixed. Similarly, when \( Y = 0 \) a.s., then (12) and (13) reduce to the optimality conditions for the two-sided market problem studied in Parker and van Alstyne [9].

Define

\[
p_c(y_0) := \frac{w - s \int_{y_0}^{\infty} f_Y(y) dy}{\int_{y_0}^{\infty} f_Y(y) dy} - p.
\]

This expression for \( p_c(y_0) \) appears in (11), when \( y_0 = y_0^* \), in which case \( p_c(y_0^*) = p_c^* \). Because we will need to use \( p_c(y_0) \) in the later part of this paper, we show below a useful property of it:

**Proposition 2** \( p_c(y_0) \) is an increasing function of \( y_0 \).

**Proof:** Note that

\[
p'_c(y_0) = \frac{(w - s)f_Y(y_0)}{\left(\int_{y_0}^{\infty} f_Y(y) dy\right)^2} > 0, \quad \text{for all } y_0.
\]

Hence, \( p_c(y_0) \) is an increasing function of \( y_0 \). 

Let

\[
\overline{q}_c := \overline{q}_c - G_c(p_c, y_0),
\]

where

\[
G_c(p_c, y_0) := H'_c(p_c) \left( (w - p_c - p)y_0 + (s - p_c - p) \int_{-\infty}^{y_0} (y - y_0) f_Y(y) dy \right).
\]

Note that we introduce \( G_c(p_c, y_0) \) to take into account changes in variance of consumer demand as \( p_c \) changes. If consumer demand variance is a constant in \( p_c \) (in which case, \( H_c \) is a constant in \( p_c \)), then \( G_c(p_c, y_0) \) is identical to zero. We take \( \overline{q}_c \) as the modified actual expected consumer demand upon taking into account the change in consumer demand variance as \( p_c \) changes.
Remark 1  Observe that
\[(w - p_c - p)y_0 + (s - p_c - p)\int_{-\infty}^{y_0} (y - y_0)f_Y(y)dy\]
\[= (w - s)\int_{-\infty}^{y_0} (y - y_0)f_Y(y)dy + (w - p_c - p)\int_{y_0}^{\infty} (y - y_0)f_Y(y)dy.\]

In the definition of \(G_c(p_c, y_0)\), since \(p_c^* \geq w - p\) (from (11), the fact that \(p(y_0) \to w - p\) as \(y_0 \to -\infty\), and Proposition 2), we have
\[(w - p_c^* - p)y_0^* + (s - p_c^* - p)\int_{-\infty}^{y_0^*} (y - y_0)f_Y(y)dy\]
\[= (w - s)\int_{-\infty}^{y_0^*} (y - y_0)f_Y(y)dy + (w - p_c^* - p)\int_{y_0^*}^{\infty} (y - y_0)f_Y(y)dy \geq 0.\]

Therefore, if \(H'_c(p_c^*) \geq 0\),
\[G_c(p_c^*, y_0^*) = H'_c(p_c^*) \left( (w - p_c^* - p)y_0^* + (s - p_c^* - p)\int_{-\infty}^{y_0^*} (y - y_0)f_Y(y)dy \right) \geq 0.\]

We now give a more useful set of conditions (which gives expressions for \(p_c^*\) and \(p_j^*\) to be used later) satisfied by \((p_c^*, p_j^*, y_0^*)\) as follows:

**Proposition 3** Let \((p_c^*, p_j^*, y_0^*)\) be an optimal solution to (9). It then satisfies
\[p_c^* = \frac{w - s \int_{-\infty}^{y_0^*} f_Y(y)dy}{\int_{y_0^*}^{\infty} f_Y(y)dy} - p,\]  
(17)
\[p_c^* = w + \frac{e_{cj}D_c(p_c^*)\overline{q}_j^* - D'_c(p_j^*)\overline{q}_c^*}{(1 - e_{cj}e_{jc})D_c(p_c^*)D'_c(p_j^*)} + \frac{H_c(p_c^*) \int_{y_0^*}^{\infty} (y - y_0)f_Y(y)dy}{(1 - e_{cj}e_{jc})D'_c(p_c^*)},\]  
(18)
\[p_j^* = \frac{D_c(p_c^*)\overline{q}_j^* - e_{jc}D'_c(p_j^*)\overline{q}_c^*}{(e_{cj}e_{jc} - 1)D_c(p_c^*)D'_c(p_j^*)} + \frac{e_{jc}H_c(p_c^*) \int_{y_0^*}^{\infty} (y - y_0)f_Y(y)dy}{(e_{cj}e_{jc} - 1)D'_c(p_c^*)}.\]  
(19)

**Proof:** Note that using the definition of \(\overline{q}_c\) in (16), (12)-(13) are equivalent to the following two conditions:
\[p_c^* = w + \frac{\left( \frac{\partial f_c}{\partial p_c} \right)^* \overline{q}_j^* - \left( \frac{\partial f_c}{\partial p_j} \right)^* \overline{q}_c^* + \left( \frac{\partial f_c}{\partial q_c} \right)^* H_c \int_{y_0^*}^{\infty} (y - y_0)f_Y(y)dy}{\left( \frac{\partial f_c}{\partial p_c} \right)^* - \left( \frac{\partial f_c}{\partial p_j} \right)^* (\overline{q}_c)},\]  
(20)
\[p_j^* = \left( \frac{\partial f_c}{\partial p_c} \right)^* \overline{q}_j^* - \left( \frac{\partial f_c}{\partial p_j} \right)^* \overline{q}_c^* + \left( \frac{\partial f_c}{\partial q_c} \right)^* H_c \int_{y_0^*}^{\infty} (y - y_0)f_Y(y)dy \right) - \left( \frac{\partial f_c}{\partial p_c} \right)^* \left( \frac{\partial f_c}{\partial p_j} \right)^* \left( \frac{\partial f_c}{\partial q_c} \right)^*.\]  
(21)

With
\[\overline{q}_c = D_c(p_c) + e_{jc}D_j(p_j),\]
\[\overline{q}_j = D_j(p_j) + e_{cj}D_c(p_c),\]
\begin{align*}
p^*_c &= w + \frac{e_{cj} D'_c(p^*_c) \bar{p}_j - D'_j(p^*_j) \bar{p}_c^*}{(1 - e_{cj} e_{jc}) D_c(p^*_c) D'_j(p^*_j)} + \frac{D'_j(p^*_j) H_c(p^*_c) \int_{y_0}^{\infty} (y - y_0) f_Y(y) dy}{(1 - e_{cj} e_{jc}) D_c(p^*_c) D'_j(p^*_j)}, \\
p^*_j &= \frac{D'_c(p^*_c) \bar{q}_j - e_{jc} D'_j(p^*_j) \bar{q}_c^*}{(e_{cj} e_{jc} - 1) D_c(p^*_c) D'_j(p^*_j)} + \frac{e_{jc} D'_j(p^*_j) H_c(p^*_c) \int_{y_0}^{\infty} (y - y_0) f_Y(y) dy}{(e_{cj} e_{jc} - 1) D_c(p^*_c) D'_j(p^*_j)},
\end{align*}

as required.

Let us now consider the case when \( e_{cj} = e_{jc} = 0 \) (that is, when there are no cross-network effects) and make an observation, which will be used later in the discussion. Suppose \((p^0_c, p^0_j, y_0)\) is the optimal solution to (9) when \( e_{cj} = e_{jc} = 0 \).

We have the following characterization of \((p^0_c, p^0_j, y_0)\):

**Proposition 4** Under Assumption D,

\begin{align*}
p_c &= p^0_c, y_0 = y_0^0 \\
\iff p_c &= w - s \frac{\int_{y_0}^{\infty} f_Y(y) dy}{\int_{y_0}^{\infty} f_Y(y) dy} - p, \quad \text{and} \\
p_c &= w - \frac{D_c(p_c) - G_c(p_c, y_0)}{D_c(p_c)} + \frac{H_c(p_c) \int_{y_0}^{\infty} (y - y_0) f_Y(y) dy}{D_c(p_c)}.
\end{align*}

Also,

\begin{align*}
p_j &= p^0_j \\
\iff p_j &= -\frac{D_j(p_j)}{D'_j(p_j)}.
\end{align*}

**Proof:** Let us consider the first “if and only if” relation.

Suppose \( p_c = p^0_c \) and \( y_0 = y_0^0 \). Then with \( p_j = p^0_j \), and \( e_{cj} = e_{jc} = 0 \),

\begin{align*}
p_c &= w - s \frac{\int_{y_0}^{\infty} f_Y(y) dy}{\int_{y_0}^{\infty} f_Y(y) dy} - p, \quad \text{and} \\
p_c &= w - \frac{D_c(p_c) - G_c(p_c, y_0)}{D_c(p_c)} + \frac{H_c(p_c) \int_{y_0}^{\infty} (y - y_0) f_Y(y) dy}{D_c(p_c)}
\end{align*}

follows from (17) and (18) in Proposition 3, since \((p^0_c, p^0_j, y_0)\) is the optimal solution to (9), when \( e_{cj} = e_{jc} = 0 \).

On the other hand, suppose

\begin{align*}
p_c &= w - \frac{\int_{y_0}^{\infty} f_Y(y) dy}{\int_{y_0}^{\infty} f_Y(y) dy} - p, \quad \text{and} \\
p_c &= w - \frac{D_c(p_c) - G_c(p_c, y_0)}{D'_c(p_c)} + \frac{H_c(p_c) \int_{y_0}^{\infty} (y - y_0) f_Y(y) dy}{D'_c(p_c)}.
\end{align*}
Let $p_j$ satisfy

$$p_j = -\frac{D_j(p_j)}{D'_j(p_j)}.$$

These are the KKT conditions for (9) when $e_{cj} = e_{jc} = 0$. Hence, by Assumption D, $p_c = p_c^0, y_0 = y_0^0$, where $(p_c^0, p_j^0, y_0^0)$ is the unique optimal solution to (9) when $e_{cj} = e_{jc} = 0$.

By similar arguments, (23) can be proven.

In what follows, $r^*$ is the ratio of spillover effects, $r$, evaluated at $(p_c^*, p_j^*)$. That is,

$$r^* = \frac{e_{cj}D'_c(p_c^*)}{e_{jc}D'_j(p_j^*)}.$$

We are now ready to introduce the key parameters $r_0^*$ and $r_1^*$ that affect optimal pricing behavior.

**Definition 1**

$$r_0^* := 1 - \frac{e_{cj}}{q_j^*} \left( H_c^* \int_{y_0^*}^{\infty} (y - y_0^*) f_Y(y) dy + G_c^* \right).$$

**Definition 2**

$$r_1^* := 1 - \frac{1}{q_c} \left( H_c^* \int_{y_0^*}^{\infty} (y - y_0^*) f_Y(y) dy + G_c^* \right).$$

**Remark 2** When $H_c(p_c) = 0$ for all $p_c$, (24) and (25) reduce to $r_0^* = r_1^* = 1$, the deterministic case considered in [9]. Our definition of $r_0^*$ and $r_1^*$ captures the deviation resulting from demand uncertainty. Furthermore, from Remark 1, if $H'_c(p_c^*) \geq 0$ (e.g., when $H_c(p_c)$ is a positive constant independent of $p_c$), then $G_c(p_c^*, y_0^*) \geq 0$, and

$$r_1^* \leq r_0^* \leq 1.$$

**Theorem 1** Let $\log D_j(p_j)$ be a concave function of $p_j$. If $e_{jc} > 0$, then

$$r^* \leq r_1^* \iff p_j^* \leq p_j^0.$$  

Equality holds on the left-hand side if and only if equality holds on the right-hand side.

If $e_{jc} < 0$, then

$$r^* \leq r_1^* \iff p_j^* \geq p_j^0.$$  

Equality holds on the left-hand side if and only if equality holds on the right-hand side.
**Proof:** We only need to show the case when \( e_{jc} > 0 \). The case when \( e_{jc} < 0 \) is similar. For the case when \( e_{jc} > 0 \), we just need to show the forward direction in (27).

(19) implies that

\[
  p_j^* = \frac{D_j(p_j^*)}{D'_j(p_j^*)} + \frac{\overline{\tau}_c^*}{e_{jc}} - 1 \left( \frac{e_{jc}}{D_j^*(p_j^*)} - \frac{e_{jc}}{\overline{D}_c^*(p_c^*)} \right) + \frac{e_{jc}G_c^*}{(e_{jc}e_{jc} - 1)\overline{D}_c^*(p_c^*)} + \frac{e_{jc}H_c^*}{(e_{jc}e_{jc} - 1)\overline{D}_c^*(p_c^*)},
\]

upon algebraic manipulations and using the definition of \( \overline{\tau}_c^* \).

Now, the sum of the last three terms in the above expression for \( p_j^* \) can be expressed the following way:

\[
  \frac{e_{jc}G_c^*}{(e_{jc}e_{jc} - 1)\overline{D}_c^*(p_c^*)} + \frac{e_{jc}H_c^*}{(e_{jc}e_{jc} - 1)\overline{D}_c^*(p_c^*)} - 1 + \frac{1}{\overline{\tau}_c^*} \left( H_c^* \int_{y_0}^{\infty} (y - y_0)f_Y(y)dy + G_c^* \right).
\]

Therefore, we have

\[
  p_j^* = \frac{D_j(p_j^*)}{D'_j(p_j^*)} + \frac{e_{jc}G_c^*}{(e_{jc}e_{jc} - 1)\overline{D}_c^*(p_c^*)} (r^* - r_1^*),
\]

using the definition of \( r^* \) and \( r_1^* \).

Suppose \( r^* < r_1^* \). Then, since \( e_{jc} > 0 \), \( \overline{\tau}_c^* > 0 \) and \( e_{jc}e_{jc} < 1 \), we have from (28)

\[
  p_j^* < \frac{D_j(p_j^*)}{D'_j(p_j^*)}.
\]

Now, suppose \( p_j^0 \leq p_j^* \). Then

\[
  \frac{D_j'(p_j^0)}{D_j(p_j^0)} \geq \frac{D_j'(p_j^*)}{D_j(p_j^*)},
\]

since \( \log D_j(p_j) \) is a concave function of \( p_j \).

From (29), we therefore have \( p_j^* < p_j^0 \).

\[\text{Note that the concavity of log } D_j(p_j) \text{ is a very mild assumption that is used quite frequently in the literature (see, for example, Rochet and Tirole [12]).} \]

We examine next the relationship between \( r^* \) and \( r_0^* \) and how it affects \( p_c^* \) and \( p_c^0 \).

**Definition 3** Let

\[
  H(y_0, p_c) := p_c - w + \frac{\overline{D}_c(p_c) - G_c(p_c, y_0)}{\overline{D}_c'(p_c)} - \frac{H_c(p_c) \int_{y_0}^{\infty} (y - y_0)f_Y(y)dy}{\overline{D}_c(p_c)}. \]
Also define

\[ H_0 := \{(y_0, p_c) : H(y_0, p_c) = 0\}, \]
\[ H_+ := \{(y_0, p_c) : H(y_0, p_c) > 0\}, \]
\[ H_- := \{(y_0, p_c) : H(y_0, p_c) < 0\}. \]

The curve \( H_0 \) partitions the \( y_0 - p_c \) plane into two disjoint portions and, by our assumptions, has only a unique intersection point with the graph of \( p_c(y_0) \), which is the point \((y_0^0, p_c^0)\) by (22). One-half of the graph of \( p_c(y_0) \) has to lie in \( H_+ \), while the other open-half of the graph lies in \( H_- \), provided that the gradient of \( H(y_0, p_c) \) is not along the same direction as the gradient of \( H_1(y_0, p_c) := p_c - p_c(y_0) \) at \((y_0^0, p_c^0)\). Making such an assumption, we formalize it as follows:

**Assumption 1**

\[ \nabla H(y_0^0, p_c^0) \neq \nabla H_1(y_0^0, p_c^0). \]

The above is a technical assumption stating that the level curve of \( H(y_0, p_c) \) passing through \((y_0^0, p_c^0)\) is not tangential at this point to the level curve of \( H_1(y_0, p_c) \) also passing through \((y_0^0, p_c^0)\).

There are two possible scenarios which we will discuss separately in the rest of this section.

### 4.1 Case 1: \((y_0, p_c(y_0)) \in H_+ \) for all \( y_0 \in (y_0^0, \infty) \)

Under Assumption 1, this case leads to \( y_0 \in (y_0^0, \infty) \Leftrightarrow (y_0, p_c(y_0)) \in H_+, \ y_0 \in (-\infty, y_0^0) \Leftrightarrow (y_0, p_c(y_0)) \in H_- \).

We have the following theorem:

**Theorem 2** Under Case 1 with Assumption 1. If \( e_{jc} > 0 \), then

\[ r^* \leq r_0^* \Leftrightarrow p_c^* \geq p_c^0. \]  

(30)

Equality holds on the left-hand side if and only if equality holds on the right-hand side.

If \( e_{jc} < 0 \), then

\[ r^* \leq r_0^* \Leftrightarrow p_c^* \leq p_c^0. \]

Equality holds on the left-hand side if and only if equality holds on the right-hand side.
Proof: As before, we just need to show the forward direction in (30) under the case $e_{jc} > 0$. The case when $e_{jc} < 0$ can be similarly proven.

(18) implies that

$$p^*_c = w - \frac{\overline{T}_c(p^*_c) - G^*_c}{\overline{D}_c(p^*_c)} + \frac{\overline{H}_c^* \int_{y_0^*}^\infty (y - y_0^*) f_Y(y) dy}{\overline{D}_c(p^*_c)}$$

$$+ \frac{\overline{q}_c}{1 - e_{cj} e_{jc}} \left( \frac{e_{cj} \overline{T}_c(p^*_c)}{\overline{D}_c(p^*_c)} - \frac{e_{jc}}{\overline{D}_c(p^*_c)} \right) + \frac{e_{cj} e_{jc} \overline{G}_c^*}{(1 - e_{cj} e_{jc}) \overline{D}_c(p^*_c)}$$

upon algebraic manipulations and using the definition of $\overline{q}_c$.

Now, the sum of the last three terms in the above expression for $p^*_c$ can be expressed the following way:

$$\frac{e_{jc} \overline{q}_c}{(1 - e_{cj} e_{jc}) \overline{D}_c(p^*_c)} \left( \frac{e_{cj} \overline{T}_c(p^*_c)}{\overline{D}_c(p^*_c)} - 1 + \frac{e_{jc}}{\overline{D}_c(p^*_c)} \left( \overline{H}_c^* \int_{y_0^*}^\infty (y - y_0^*) f_Y(y) dy + \overline{G}_c^* \right) \right).$$

Therefore, we have

$$p^*_c = w - \frac{\overline{T}_c(p^*_c) - G^*_c}{\overline{D}_c(p^*_c)} + \frac{\overline{H}_c^* \int_{y_0^*}^\infty (y - y_0^*) f_Y(y) dy}{\overline{D}_c(p^*_c)} + \frac{e_{jc} \overline{q}_c}{(1 - e_{cj} e_{jc}) \overline{D}_c(p^*_c)} (r^* - r_0^*), \quad (31)$$

using the definition of $r^*$ and $r_0^*$.

Suppose $r^* < r_0^*$.

Then, using (31), $e_{jc} > 0$, $\overline{q}_c > 0$ and $e_{cj} e_{jc} < 1$, we have

$$p^*_c > w - \frac{\overline{T}_c(p^*_c) - G^*_c(p^*_c, y_0^*)}{\overline{D}_c(p^*_c)} + \frac{\overline{H}_c(p^*_c) \int_{y_0^*}^\infty (y - y_0^*) f_Y(y) dy}{\overline{D}_c(p^*_c)}. \quad (32)$$

Therefore, we have $H(y_0^*, p^*_c) = H(y_0^*, p_c(y_0^*)) > 0$, that is, $(y_0^*, p_c(y_0^*)) \in H_+$, which implies that $y_0^* > y_0^0$. Hence $p^*_c > p_0^*$, by Proposition 2.

Next, we show that when demand is additive, and under mild technical conditions, Case 1 holds:

Theorem 3 Suppose $H_c(p_c) = \text{constant}$. If $e_{jc} > 0$, and the expected profit from the consumer side of the two-sided market, $(p_c - w)\overline{q}_c$, decreases as $p_c$ increases for large $p_c$ and fixed $p_j$, then Case 1 holds.
Proof: Since $H_c(p_c) = \text{constant} = k$, we have $G_c(p_c, y_0) = 0$. Hence,

$$H(y_0, p_c) = p_c - w + \frac{D_c(p_c) - G_c(p_c, y_0)}{D_c'(p_c)} - \frac{H_c(p_c) \int_{y_0}^{\infty} (y - y_0) f_Y(y) dy}{D_c'(p_c)}$$

$$= p_c - w + \frac{D_c(p_c)}{D_c'(p_c)} = \frac{k \int_{y_0}^{\infty} (y - y_0) f_Y(y) dy}{D_c'(p_c)}.$$

Under the assumption that $(p_c - w)\eta_c$ decreases for large $p_c$ and fixed $p_j$, we have

$$\eta_c + (p_c - w)D_c'(p_c) < 0,$$

which implies that

$$D_c(p_c) + (p_c - w)D_c'(p_c) < 0$$

for large $p_c$, since $e_j D_j(p_j) \geq 0$. That is,

$$1 + \frac{D_c(p_c)}{(p_c - w)D_c'(p_c)} > 0$$

for large $p_c$.

From (33),

$$H(y_0, p_c) = (p_c - w) \left( 1 + \frac{D_c(p_c)}{(p_c - w)D_c'(p_c)} \right) - \frac{k \int_{y_0}^{\infty} (y - y_0) f_Y(y) dy}{D_c'(p_c)} > 0$$

for large $p_c$.

Therefore, Case 1 holds for the additive demand.

Example 1 Suppose

$$D_c = 10 - 0.1 p_c, \ D_j = 10 - 0.1 p_j, \ \text{and} \ H_c(p_c) = 1.$$

Y follows a distribution with density distribution function

$$f_Y(y) = \begin{cases} 0.5 \exp(-y), & y \geq 0; \\ 0.5 \exp(y), & y < 0. \end{cases}$$

The unit production cost $w = $20, salvage cost $s = $5, and penalty cost $p = $5.

It is easy to calculate

$$H(y_0, p_c(y_0)) = \begin{cases} 60e^{y_0} + 5e^{-y_0} - 120, & y_0 \geq 0; \\ \frac{40 - 5e^{y_0}}{1 - 0.5e^{y_0}} + 5e^{y_0} - 10y - 130, & y_0 < 0. \end{cases}$$
As shown in Figure 1, \( H(y_0, p_c(y_0)) \) crosses the x axis when \( y_0 = y_0^0 = 0.671 \). It is thus easily seen that this example belongs to Case 1, that is, if \( y_0^* > y_0^0 \), \( H(y_0^*, p_c^*) > 0 \), and if \( y_0^* < y_0^0 \), \( H(y_0^*, p_c^*) < 0 \).

We can easily vary the values of \( e_{cj} \) and \( e_{jc} \) to obtain different pricing strategies.

- If \( e_{cj} = 0.5, e_{jc} = 0.49 \), \( p_c^*(\$58.77) > p_c^0(\$58.76), p_j^*(\$50.41) > p_j^0(\$50), y_0^0(0.6724) > y_0^0(0.6721) \) and \( r_1^*(0.961) < r^*(0.98) < r_0^*(0.982) \). The platform intermediary would charge a surplus in both the consumer and the content provider markets.

- If \( e_{cj} = 0.5, e_{jc} = 0.4 \), \( p_c^*(\$62.76) > p_c^0(\$58.76), p_j^*(\$46.76) < p_j^0(\$50), y_0^0(0.73) > y_0^0(0.6721) \), and \( r^*(0.8) < r_1^*(0.963) < r_0^*(0.986) \). The platform intermediary would surcharge in the consumer market and subsidize in the content provider market.

- If \( e_{cj} = 0.5, e_{jc} = 0.8 \), \( p_c^*(\$37.72) < p_c^0(\$58.76), p_j^*(\$71.13) > p_j^0(\$50), y_0^0(0.20) < y_0^0(0.6721) \), and \( r_1^*(0.947) < r_0^*(0.958) < r^*(1.6) \). The platform intermediary would subsidize in the consumer market and surcharge in the content provider market.

### 4.2 Case 2: \( (y_0, p_c(y_0)) \in H_- \) for all \( y_0 \in (y_0^0, \infty) \)

Under Assumption 1, this case leads to \( y_0 \in (y_0^0, \infty) \iff (y_0, p_c(y_0)) \in H_- \), \( y_0 \in (-\infty, y_0^0) \iff (y_0, p_c(y_0)) \in H_+ \).

We have the following theorem:
Theorem 4 Under Case 2 with Assumption 1. If $e_{jc} > 0$, then

$$r^* \leq r_0^* \iff p_c^* \leq p_c^0.$$  \hspace{1cm} (34)

Equality holds on the left-hand side if and only if equality holds on the right-hand side.

If $e_{jc} < 0$, then

$$r^* \leq r_0^* \iff p_c^* \geq p_c^0.$$  \hspace{1cm} (35)

Equality holds on the left-hand side if and only if equality holds on the right-hand side.

Proof: As before, we just need to show the forward direction in (34), under the case when $e_{jc} > 0$.

Writing (31) here again,

$$p_c^* = w - \frac{D_c(p_c^*) - G_c(p_c^*, y_0^*)}{D_c(p_c^*)} + \frac{H_c(p_c^*) \int_{y_0^*}^{\infty} (y - y_0^*) f_Y(y)dy}{D_c(p_c^*)} + \frac{e_{jc} \overline{r}_j^*(r^* - r_0^*)}{(1 - e_{cj} e_{jc}) D_c(p_c^*)}.$$  \hspace{1cm} (35)

Suppose $r^* > r_0^*$.

Then, using (35), $e_{jc} > 0$, $\overline{r}_j^* > 0$ and $e_{cj} e_{jc} < 1$, we have

$$p_c^* < w - \frac{D_c(p_c^*) - G_c(p_c^*, y_0^*)}{D_c(p_c^*)} + \frac{H_c(p_c^*) \int_{y_0^*}^{\infty} (y - y_0^*) f_Y(y)dy}{D_c(p_c^*)}.$$  \hspace{1cm} (36)

Therefore, we have $H(y_0^*, p_c^*) = H(y_0^*, p_c(y_0^*)) < 0$, that is, $(y_0^*, p_c(y_0^*)) \in H_-$, which implies that $y_0^* > y_0^0$. Hence, $p_c^* > p_c^0$, by Proposition 2.

Example 2 Suppose $D_c = 50 + \exp(-0.5 p_c)$, $D_j = 50 - 0.5 p_j$, and $H_c(p_c) = \exp(-0.5 p_c)$. $Y$ follows a distribution with density distribution function

$$f_Y(y) = \begin{cases} 0.5 \exp(-y), & y \geq 0; \\ 0.5 \exp(y), & y < 0. \end{cases}$$

The unit production cost $w = $15, salvage cost $s = $14, and penalty cost $p = $14.

As shown in Figure 2, $H_0$ and the graph of $p_c(y_0)$ cross when $y_0 = y_0^0(-90.45)$. It is thus easy to see that this example belongs to Case 2, that is, if $y_0^* > y_0^0$, $H(y_0^*, p_c^*) < 0$, and if $y_0^* < y_0^0$, $H(y_0^*, p_c^*) > 0$.

We thus could vary $e_{cj}$ and $e_{jc}$ to obtain different pricing strategies.
Figure 2: $H_0$ and $p_c(y_0)$ of Example 2

- If $e_{cj} = 0.7$ and $e_{jc} = 0.5$, $p_c^*(1) \geq p_c^0(1)$, $p_j^0(88.92) > p_j^0(50)$, $y_0^0(-63.87) > y_0^0(-90.45)$, and $r_1^*(0.27) < r_0^*(0.34) < r^*(0.85)$. The platform intermediary would surcharge in both the consumer and the content provider markets.

- If $e_{cj} = 0.1$ and $e_{jc} = 0.9$, $p_c^*(1) \leq p_c^0(1)$, $p_j^0(61.36) > p_j^0(50)$, $y_0^*(116.04) < y_0^0(-90.45)$, and $r_1^*(0.35) < r^*(0.07) < r_0^*(0.71)$. The platform intermediary would subsidize in the consumer market and surcharge in the content provider market.

4.3 Summary

Note that Case 1 and Case 2 are mutually exclusive. Given a platform intermediary with a two-sided market, to determine whether it belongs to Case 1 or Case 2, it is enough and not difficult to choose a point $y_0 > y_0^0$ and to find out whether $(y_0, p_c(y_0)) \in H_+$ (in which case it belongs to Case 1) or $H_-$ (in which case it belongs to Case 2).

Theorems 1, 2, and 4 provide generalizations of similar results in [9]. Instead of the value 1 in [9], we have in the generalized situation, $r_0^*$ and $r_1^*$, which are less than or equal to 1 and $r_1^* \leq r_0^*$, if $H'_c(p_c^*) \geq 0$. In general, these values differ. Our results indicate that if $r_1^* < r_0^*$, and if $r^*$ lies between $r_1^*$ and $r_0^*$, then it is possible to charge both sides less than the base selling prices in the case when $e_{jc} < 0$, and charge both sides more than the base selling prices in the case when $e_{jc} > 0$. This situation of charging both sides more (or less) at the same time cannot be reflected in the model of [9]. Figure 3 below illustrates a scenario.
Table 1: Subsidy Schemes for Different Internetwork Externality Terms.

<table>
<thead>
<tr>
<th></th>
<th>$e_{jc} &gt; 0, e_{cj} \geq 0$</th>
<th>$e_{jc} &lt; 0, e_{cj} \geq 0$</th>
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</thead>
<tbody>
<tr>
<td>$p_c$</td>
<td></td>
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<tr>
<td>Case 1</td>
<td>$r^* &lt; r_0^* \iff p_c^* &gt; p_c^0$</td>
<td>$r^* &lt; r_0^* \iff p_c^* &lt; p_c^0$</td>
</tr>
<tr>
<td>Case 2</td>
<td>$r^* &lt; r_0^* \iff p_c^* &lt; p_c^0$</td>
<td>$r^* &lt; r_0^* \iff p_c^* &gt; p_c^0$</td>
</tr>
<tr>
<td>$p_j$</td>
<td></td>
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</tr>
<tr>
<td>$r^* &lt; r_1^* \iff p_j^* &gt; p_j^0$</td>
<td>$r^* &lt; r_1^* \iff p_j^* &lt; p_j^0$</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Subsidy Schemes for Different Internetwork Externality Terms.

As shown in Table 1, the subsidy direction depends on whether $e_{jc}$ is positive or negative. The subsidy direction reverses when $e_{jc}$ changes sign.

Remark 3 In the degenerate case when $e_{jc} = 0$ and $e_{cj} > 0$, if $e_{jc}$ approaches zero from the positive side, then $p_j^* > p_j^0$, while in Case 1, $p_c^* < p_c^0$, and in Case 2, $p_c^* > p_c^0$. If $e_{jc}$ approaches zero from the negative side, then $p_j^* < p_j^0$, while in Case 1, $p_c^* > p_c^0$, and in Case 2, $p_c^* < p_c^0$.

5 Numerical Studies

5.1 Sensitivity analysis

Consider the base model given as follows:

There are two independent markets (i.e., $e_{cj} = e_{jc} = 0$). The consumer market faces demand $D_c = A_c - b_c p_c + Y$ with $Y$ following a uniform distribution $U[-10, 10]$, unit production cost $w = 2$, salvage cost $s = 1$, and penalty cost $p = 1$. The content provider market faces demand
\[ D_j = A_j - b_j p_j. \] Here we let \( A_c = A_j = A(100), b_c = b_j = b(0.5). \)

The optimal solution of the base model (as well as its variants) can be obtained by solving the corresponding KKT conditions as follows:

\[
\begin{align*}
p'_c &= \frac{20w - 9y'_0 - 10s}{10 - y'_0} - p, \\
2bp'_c + (e_{cj} + e_{jc})bp'_j + \frac{(y'_0)^2}{20} - 2.5 + bw - (1 + e_{jc})A &= 0, (37) \\
(e_{cj} + e_{jc})bp'_c + 2bp'_j - e_{jc}bw - (1 + e_{cj})A &= 0.
\end{align*}
\]

Usually there are multiple roots to (37). The solution satisfying Assumption C and with the highest profit is selected as the optimal solution \((p'_c, p'_j, y'_0)\).

We obtain variants of the base model by slightly varying the parameters: (i) internetwork externality (i.e., \( e_{jc} \) and \( e_{cj} \)); (ii) demand elasticity (i.e., \( b_c \) and \( b_j \)); and (iii) production cost (i.e., \( w \)) of the basic model to study the impact of their changes. We focus on exploring the sensitivity of profit, price, expected demand, and production quantity in both markets to changes in these parameters.

**Study 1: Internetwork externality.**

We consider the following three variants of the base model: (1) \( e_{cj} = e_{jc} = e \), where \( e = 0, 0.1 \ldots 0.9 \); (2) \( e_{cj} = 0.5, e_{jc} = e \), where \( e = -0.9, -0.8 \ldots 0.9 \); and (3) \( e_{cj} = e, e_{jc} = -e \), where \( e = 0, 0.1 \ldots 0.9 \). Figures 4 to 6 show the relationships between \( e_{jc} \) (or \( e_{cj} \)) and the profits, prices, expected demand quantities, and \( r^*, r^*_0, r^*_1 \) in variants (1) to (3), respectively.

Several observations can be obtained from the study. First, \( r^*_0 \) and \( r^*_1 \) change very little when \( e_{jc} \) and \( e_{cj} \) change. This result suggests that \( r^*_0 \) and \( r^*_1 \) are good indicators to the relationship of \( p'_c/p'_j \) and \( p^0_0/p^0_j \), but cannot measure the gap between them.

Second, the system still benefits even if \( e_{jc} \) is negative. As shown in Figure 6, where \( e_{jc} = -e \) and \( e_{cj} = e \), the content provider market has a negative internetwork externality factor, that is, an increase in the content provider market will hurt the consumer market. This situation can be seen in the portal and media industries.

The system’s profits are still higher than those of the independent markets from subsidies for the consumers and surcharges on the content providers. This seems to indicate that the benefit of a positive \( e_{cj}(= e) \) always covers the cost of the negative \( e_{jc} (= -e) \), and that the gap between them will increase as \( e \) increases.
Figure 4: $e_{jc} = e_{cj} = e (r^* = 1)$.

Figure 5: $e_{cj} = 0.5, e_{jc} = e$. 
Study 2: Demand elasticity.

This section studies the impact of demand elasticity on profits, prices, and demand. We fix \( e_{jc} = e_{cj} = 0.5 \) and focus on the impact of demand elasticity of the consumer market demand function (i.e., \( D_c = A - b_c p_c + Y \)). Specifically we analyze the following variant:

\[
D_c = 100 - b_c p_c + Y \quad \text{and} \quad D_j = 100 - 0.8 p_j, 
\]

where \( b_c = 0.1, 0.2 \ldots 1.9 \); the result is shown in Figure 7. Note we discuss only the demand elasticity in the consumer market here because of the very similar result obtained from the analysis of the content provider market.
As Figure 7 shows, the system profit increases if the consumer market becomes inelastic. The increase in the system profit is obtained by subsiding the more elastic market (i.e., the content provider market) and gaining extra profit from the inelastic market (i.e., the consumer market). Especially when the consumer market is very inelastic (e.g., \( b_c = 0.1 \) or \( 0.2 \) ), the system would probably obtain a significant increase in profit via charging the inelastic consumer market a very high price, subsidizing the elastic content provider market with even a negative price to increase the actual demand in both markets, and grasping more profit from the inelastic consumer market. According to microeconomics theories, demand in an inelastic (elastic) market is insensitive (sensitive) to price, and thus it is reasonable to charge a high price in the inelastic market because the demand would decrease more slowly, and charge a low price in the elastic market because demand would increase more quickly; the system would thus sell more products at the high price level.

**Study 3: Production cost.**

The impact of production cost on price and profit is a hot topic even in single-sided markets. Raz and Porteus [11] constructed an example using fractiles to represent real demand and showed that optimal retailing price is not monotonic increasing (or decreasing) in production cost. In this section, we partly adopt Example 2 in their paper to investigate the relationship between optimal retailing price \( p^*_c \) and production cost \( w \) in our two-sided market.

For the content provider market, we assume \( D_j = 107.8545 - 3.4865p_j \). For the consumer market, salvage cost \( s = 2 \) and penalty cost \( p = 5 \), \( D_c = 107.8545 - 3.4865p_c + H_c(p_c)Y \). \( Y \) follows a distribution with a density distribution function

\[
f(y) = \frac{y^2}{2l^3/3}, \quad y \in [-l, l] \quad l = 10.
\]

\[
H_c(p_c) = \frac{26.7719}{l\sqrt{0.6}} + \frac{1.585}{l\sqrt{0.6}p_c}.
\]

We further take \( e_{cj} = 0.48 \) and \( e_{jc} = 0.5 \).

As shown in Figure 8, the optimal price \( p^*_c \) first increases in \( w \), and decreases in \( w \) when \( w \) is larger than about 10. Thus our results support the non-monotonic pricing phenomenon even in the two-sided market setting.
Figure 8: Price is non-monotonic in $w$.

Figure 9: The benefit of considering spillover factors when making decisions.
5.2 The benefit of considering spillover factors

In this subsection, we investigate the benefit of considering the relationship between the consumer market and content provider market by regarding them as a whole when making decisions. We are interested to know what the possible profit loss is if a manager considers these two markets as independent and isolated when making decisions.

Suppose in reality \( e_{jc} \neq 0 \) and/or \( e_{cj} \neq 0 \). Let \( \pi_1 \) represent the profit obtained by setting price \( p_c = p^0_c, p_j = p^0_j \) and producing \( x^0 \) because the manager thinks that \( e_{jc} = e_{cj} = 0 \). Figure 9 plots the ratio of \( \pi_1 \) to \( \pi^* \) in variant (1) and (3). If both \( e_{jc} \) and \( e_{cj} \) are positive, the decision to ignore the spillover effects would cost the company up to 20% in profits. If \( e_{jc} \) is negative, the loss would increase up to around 45%.

6 Conclusions

In this paper, we investigate the impact of demand uncertainty on the optimal pricing strategy in a two-sided market. We embed the newsvendor pricing model in the console market, and examine the impact of internetwork externalities on the optimal pricing and inventory strategies in the system. This study improves upon the economic model proposed in Parker and van Alstyne [9], which derives a strong prediction for the optimal pricing behavior - one side of the market must be a profit center and the other side either a loss leader or neutral. This prediction does not explain the pricing patterns observed in some two-sided markets, where both sides are viewed as profit centers (e.g., some traditional newspaper markets and high-end fashion magazines). By incorporating the impact of supply chain operational costs, we augment and improve the economic model. More importantly, our analysis illuminates the importance of the two parameters \( r^*_0 \) and \( r^*_1 \) vis-a-vis the spillover ratio \( r^* \), and how these parameters can be used to completely characterize the different regimes in the optimal pricing solutions.

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References


