Liquidity Risk and Limited Arbitrage:  
Why Banks Lend to Opaque Hedge Funds

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Abstract

Hedge funds attempting to take advantage of market-wide liquidity shocks are not limited by opaqueness-caused capital constraints, because banks optimally supply them with backup credit lines. During such shocks, government-protected bank deposits receive inflows that lower a bank’s opportunity cost of lending. The inflows also provide a private signal that helps the bank estimate the timing and magnitude of a shock, thus reducing the information asymmetry constraining hedge funds. The unique combination of exclusive low funding cost and sophisticated information gives banks an advantage in lending to hedge funds.

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Introduction

At the opposite regulatory extremes, banks and hedge funds are key suppliers of liquidity, linked by a rapidly growing business. Recent papers by Berger and Bouwman (2007), Gatev and Strahan (2006), Kashyap et al. (2002) and others have documented bank-supplied liquidity. Evidence that hedge funds supply liquidity can be found in Getmansky et al. (2004), Hasanhodzic and Lo (2007) and Chan et al. (2007), who document significant hedge fund exposures to illiquidity risk. The Long Term Capital Management (LTCM) 1998 episode demonstrated dramatically how the inherent liquidity risk exposures of banks can be compounded by their exposures to hedge fund counter-parties. A number of papers like Greenspan (2005), Garbaravicius and Dierick (2005) and others\(^\text{1}\) imply that understanding how banks are linked to hedge funds is important for financial system stability. Despite this, little is known about the liquidity risk-sharing between hedge funds and banks. This paper develops an equilibrium model of bank lending to opaque hedge funds where bank deposit inflows help banks distinguish between systematic liquidity shocks and idiosyncratic credit risk events. Together with the exclusive low cost funding from deposits, this sophisticated information enables banks to lend to opaque hedge funds. In this way opportunistic hedge funds avoid the capital constraints that supposedly “limit arbitrage”\(^\text{2}\) due to information asymmetry.

While capital invested in hedge funds has ballooned recently, the volume of large bank prime brokerage business with hedge funds also has grown enormously.\(^\text{3}\) With a multitude of hedge funds, bank exposures to hedge funds have become better diversified,\(^\text{4}\) reestablishing the primary importance of systematic liquidity shocks rather than individual hedge fund failures. The recent collapse of the 9.2 billion dollar hedge fund Amaranth in the middle of September 2006 illustrates how banks had limited their exposures to Amaranth to a much lower level compared to LTCM.

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\(^{1}\)See also Chan et al. (2007), Edwards (1999), Kho et al. (2000), the ECB November 2005 report and their references.

\(^{2}\)Limited arbitrage refers to the adverse effect of capital flight during transitory price shocks, that is due to asymmetric information. See Shleifer and Vishny (1997) and below.

\(^{3}\)Recent survey data show that most large banks offer prime brokers services to hedge funds and that this bank revenue stream is the one that is growing most rapidly. See ECB November 2005 report for findings and further details about the business relationship between banks and hedge funds.

\(^{4}\)Diamond (1984) argues that the role of delegated monitor explains why banks tend to hold large and well diversified loan portfolios and fund themselves primarily with debt.
Nevertheless, given that hedge funds opaqueness is considered essential by their operators, the question that remains open is how banks are able to lend to such opaque borrowers.

Why do banks lend to hedge funds? Gatev and Strahan (2006) show that banks are uniquely endowed with a generic hedge for systematic liquidity risk. The hedging exposure stems from offsetting inflows into government-protected bank deposits. If hedge funds were transparent to banks, these inflows would allow banks to sell their excess liquidity to the funds at the precise times when they are eager to "double-down"\textsuperscript{5}, but are unable to raise new capital, because their uninformed investors interpret transitory losses as indications of low hedge fund quality.\textsuperscript{6} However, in order to lend to hedge funds, banks also have to overcome the same information asymmetry, arising from hedge fund opaqueness, that confounds other hedge fund investors. In this paper we argue that banks are uniquely endowed with such capacity due to the private signal provided by deposit inflows.

Fama (1985) has argued that private information from business checking accounts gives banks an advantage in lending over other intermediaries.\textsuperscript{7} This bank advantage is reduced dramatically in the case of hedge funds, because they are not required to disclose their investment strategies and, in fact, actively try to conceal them in order to remain competitive. This paper argues that the aggregate transaction deposit franchise has another important informational role that mirrors Fama’s notion of access to private borrower-specific information: aggregate deposit inflows help banks estimate market-wide events and enable them to lend to opaque borrowers. This explains why prime brokers in fact do supply liquidity backup lines that hedge funds use during liquidity shocks.\textsuperscript{8}

\textsuperscript{5}This gambling term means doubling the bet when the expected return is high, and is appropriate in the context of hedge funds, whose incentives induce high risk-taking.

\textsuperscript{6}This argument mirrors the notion of Shleifer and Vishny (1997), that capital is withdrawn when investors, who do not have access to insider information, interpret poor past investment performance as reflection of poor management quality. Recent contracting innovations, like long redemption notice periods, have removed the Shleifer-Vishny constraint, but not the "double-down" problem.

\textsuperscript{7}Mester et al. (2006) offer empirical evidence for this notion.

\textsuperscript{8}Lending to hedge funds by deposit taking institutions may involve another layer of intermediation – investment bank prime brokers, who purchase liquidity backup lines from commercial banks and then offer liquidity insurance to hedge funds. Since the 1999 repeal of the Glass-Steagall Act, most large banks have become broad financial
The intuition behind our model is that banks are able to offer “bridge” liquidity lines and lend during systematic liquidity shocks because their transaction deposits inflows provide additional information useful in estimating a market-wide liquidity shock that cause opportunistic hedge funds to draw down their backup lines. Banks with large transaction deposit franchises experience massive deposits inflows during short-term market-wide flights to quality, and such inflows signal the precise timing and help estimate the size of a systematic liquidity shock. In this way, banks not only have the exclusive capacity to supply liquidity, but also have the ability to better evaluate the extent to which hedge fund performance and demand for liquidity are due to exposure to a systematic liquidity shock as opposed to idiosyncratic distress due to poor investment decisions.

The intuition of this argument is easily followed if one relies on the standard representation of equity in a levered entity as an option contract. A bank that sells liquidity insurance effectively writes a put option on the illiquid hedge fund portfolio. The portfolio is illiquid in the sense that the bank does not know its precise composition and hence is not able to hedge the put option by dynamically rebalancing a replicating portfolio in the usual way. Thus, the bank prices the option it writes not by the absence of arbitrage, but in a competitive equilibrium, using the bank’s own price of hedge-fund-specific risk. The value of the option then depends on the bank’s estimate of the time-varying drift of the underlying hedge fund portfolio, as well as on the time-varying opportunity cost of bank lending, which determines to the bank’s price of risk. The main tradeoff is that higher ex-ante liquidity risk exposure of the hedge fund makes the option more valuable, but a large liquidity shock realization also increases the conditional expected return of the asset and lowers the opportunity cost of the bank and the required liquidity risk premium for the option, making the option less valuable.

The size of the bank’s transaction deposit franchise affect this option valuation in two ways that correspond to the two roles played by the deposits. First, increasing the deposit base implies larger inflows that lower the opportunity cost of lending and hence lower the price of risk that the bank charges in equilibrium. Second, better diversification of the inflows in a larger deposit base provides a more precise signal about market-wide flight-to-quality shocks that are likely to be reversed in the short run. 

*Robert Merton, Ceteris paribus, the inflows of banks with larger transaction deposit franchises have institutions that engage in both deposit taking and in investment banking and brokerage.*
higher correlation with a systemic shock and imply a higher conditional estimate of the drift. This drift increase makes it more likely that the option will expire out of the money. Both effects lower the value of the option written by the bank. Since the upside for the bank is the interest it charges, the lower option valuation is translated into an optimal lending increase and higher bank profits. As a result, banks with large transaction deposit franchises optimally lend to hedge funds more than other lenders less deposits.

The model produces several new empirical implications. First, banks with larger transaction deposits relative to other banks, are better at reducing the information asymmetry about hedge funds and hence such banks ex-ante would optimally offer more liquidity backup lines to hedge funds. Next, the model implies that banks with larger transaction deposits lend more to hedge funds during systematic liquidity shocks, possibly lowering margin requirements. Conversely, hedge funds optimally choose to buy liquidity insurance from banks with large transaction deposit franchises, because the higher quality of such banks’ signals about the liquidity shock implies that they are more likely to facilitate borrowing of ”bridge liquidity” through lowering of margins. Finally, the model predicts that opportunistic hedge funds with backup liquidity lines from banks with large transaction deposit franchises on average outperform peers who are unable to avail themselves of the lower-quality liquidity insurance offered by banks with less transaction deposits.

Our analysis also can be helpful in developing policy implications. While a careful analysis of policy implications is beyond the scope this paper, a simple illustration is provided in the hope of stimulating future research. Its main objective is to highlight the intuition that if the recent advent of hedge funds has changed the allocations of costs and benefits for different types of agents, then the particular structure of the government safety net of the past may no longer be optimal in the sense of maximizing social surplus.

The paper is organized as follows. Section one provides a brief background on the institutional details of the banks-hedge fund business relationship. In section two, some new empirical evidence on hedge funds is presented to motivate the model. Section two presents a model of liquidity risk sharing through liquidity backup lines offered by banks to hedge funds and discusses empirical implications. Section four offers an illustrative welfare analysis. Section five concludes.
I. Background

I.1 The Hedge Fund Perspective

While hedge fund investment strategies cover the broadest universe possible, most hedge funds typically use leverage to enhance profitability. A number of hedge fund styles, notably those involving fixed income securities, profit from small spreads that yield meaningful absolute returns only with use of substantial leverage. Banks are the main source of credit to hedge funds.

One important reason why hedge funds typically borrow from banks is that only banks have enough funds to lend. Hedge funds typically try to borrow very large amounts over short periods of time and only banks that fund themselves with deposits can supply the large amounts demanded on short notice. As a result, hedge funds rely on banks’ prime brokerage for their borrowing needs, with larger hedge funds often dealing with several prime brokers in order to ensure availability and competitive pricing.

Some hedge funds specialize in holding illiquid assets and providing liquidity by actively trading in these assets, effectively acting as market-makers when liquidity is otherwise low (see Getmansky et al. (2004) and Hasanhodzic and Lo (2007)). This may allow a hedge fund to pick-up an "illiquidity" discount, as the hedge fund has longer investment horizon. Hedge funds face a potential problem if they wish to borrow in order to take advantage of a systematic liquidity shock that also has depressed the hedge fund’s net asset value. In such scenarios, the information asymmetry between the fund and its financiers may cause performance-based allocation of capital. If the bank is unable to identify a temporary liquidity shock that depresses the fund’s net asset value and attributes poor performance to low quality of the hedge fund management, then the bank may ration lending, increase margins and require collateral transfer. Credit rationing is especially important in the case of short-term lending and high quality cash collateral. In the case of systematic liquidity shocks, such performance-based credit rationing problem can be solved by backup liquidity lines extended by the bank. Hedge funds are able to draw on such lines when banks can distinguish transitory systematic liquidity shocks from fund-specific under-performance due to low skill.

\footnotetext{9This scenario mirrors the equity withdrawals of performance-based arbitrage in Shleifer and Vishny (1997). Note however that lock-up and redemption notice provisions have largely resolved the Shleifer-Vishny withdrawal conundrum, leaving open its complementary problem, where hedge funds are unable to raise additional capital.}
Backup credit lines are very important for hedge funds that face financial constraints when there are opportunities for new profitable investments. For example, if the market temporarily moves against the fund position, followed by a reversal, the hedge fund would garner additional profit by increasing the position in the interim. However, the hedge fund may not be able to attract additional equity investment on short notice, due to information asymmetry. Hedge funds reserve backup credit lines in order to be able to invest when no other financing is available. Thus, hedge funds often depend on such reserve liquidity for their most profitable investments. Such high profits are especially important to the hedge fund operators, due to the hedge fund incentive structure. As hedge fund principals are compensated with a substantial fraction of the profit, higher absolute profits are important to them beyond establishing track-record and attracting new capital.

I.2 The Bank Perspective

Banks engage in two principal types of business dealings with hedge funds: financing and trading.\textsuperscript{10} The banks with the largest volume of business with hedge funds are prime brokers that lend cash and securities to hedge funds whose investment strategies rely on leverage and short-selling. Most of the bank lending is short term and typically is fully collateralized, or over-collateralized, with high quality (typically cash) collateral.

Banks also profit from their trading with hedge funds, who often trade heavily or are the main participant in otherwise less liquid markets. While hedge funds typically focus on order size and execution, the pricing and spreads is typically of secondary importance to them and thus offer attractive margins to banks.\textsuperscript{11} When hedge funds trade with banks’ trading desks, private information of a hedge fund is revealed to the bank through the hedge fund’s trades.\textsuperscript{12} The bank trading desks then use this information to mimic hedge fund strategies that have high expected profitability.

\textsuperscript{10}In addition, many banks invest in hedge funds. We do not focus on this activity, where the bank has the role of a passive client.

\textsuperscript{11}Hedge funds typically operate with outside capital and have performance-based incentives that induce substantial risk-taking and make shopping for the best price of execution services a second-order consideration. See discussion below.

\textsuperscript{12}This is an example of the notion proposed by Fama (1985) that banks obtain private information from observing borrower transactions.
Evidence from a bank survey presented in the European Central Bank (ECB) report from November 2005 shows that hedge fund demand for this type of product is growing rapidly. One very important element of bank financing for hedge funds is backup, or "bridge" liquidity lines that allow hedge funds to take advantage of new investment opportunities when raising new equity is not feasible due to information asymmetry between the hedge fund managers and investors.\textsuperscript{13} Banks are the low cost liquidity providers\textsuperscript{14}, while hedge funds are opaque and face capital constraints during liquidity shocks, so there is scope for efficiency gains in banks-hedge funds risk-sharing of systematic liquidity risk. Gatev and Strahan (2006) show that banks have an advantage in hedging systematic liquidity risk, because their transaction deposits, protected by FDIC insurance and an implicit government safety net,\textsuperscript{15} receive liquidity during short-term flights to quality. In particular, they show that during periods of market stress, when nervous investors pull out of asset markets and depress prices, banks experience inflows into their transaction deposits as investors deposit their funds in the banks, perceived as a safe haven. This provides extra liquidity exactly at the time when hedge funds would require it. Multiple hedge funds are able to draw large amounts on their backup lines, without creating a liquidity shortage for the bank, which does not keep liquidity reserves that are large enough to satisfy demand by hedge funds.

The effective systematic liquidity risk exposure of a bank can be very significant, when the banks have a large exposure to a single, or few, hedge funds with high exposures. The development of the LTCM crisis revealed the fragility of the banking system, which had substantial non-diversified exposure to systematic liquidity risk via large, non-diversified exposures to LTCM. The bank problem was exacerbated by under-collateralization resulting from LTCM’s borrowing from multiple lenders. In effect, LTCM, had diversified its extreme liquidity risk exposure among banks with their implicitly guaranteed bail-out. LTCM’s losses from the systematic liquidity risk shock real-

\textsuperscript{13}As discussed above, the availability of these credit lines is important for hedge funds who potentially profit most from investment opportunities that coincide with temporary shocks to liquidity in their respective markets. Backup liquidity may also mitigate the short-term effect of larger redemptions. More generally, liquidity backup lines, along with lock-ups and other provisions, help avoid realizing losses in times of short-term market stress.

\textsuperscript{14}For the role of banks as liquidity providers, see Diamond and Dybvig (1983), Garber and Weisbrod (1990), Myers and Rajan (1998), Diamond and Rajan (2001), Kashyap et al. (2005), Berger and Bouwman (2005), Gatev et al. (2005), Gatev and Strahan (2006), Gatev et al. (2006) and their references.

\textsuperscript{15}See for example, O’Hara and Shaw, (1990) and Pennacchi (2005)
ization ultimately were absorbed by the banking system. The ex-post profitability of the liquidated LTCM positions after the crisis has highlighted the importance of the banks’ ability to identify systematic liquidity shocks as transitory.

This paper argues that bank deposit inflows that are inversely correlated with liquidity shocks not only lower the opportunity cost of banks, but also provide banks with information about systematic liquidity shocks. High correlation between deposit inflows and systematic liquidity shocks helps banks evaluate the magnitude of a shock, reducing the information asymmetry between them and hedge funds that are exposed to the shock and would like to borrow in anticipation of its reversal.

The combination of low cost funding from deposits and information during a systematic liquidity shock is available only to banks protected by the government safety net. This combination gives banks an advantage over other lenders in the business of lending to hedge funds. To the extent that large banks’ lending to hedge funds is competitive, the bank advantage is competed away to the benefit of hedge funds. The bank capacity to bear systematic liquidity risk is efficiently shared with hedge funds, who transfer their systematic liquidity risk exposure to the banking system at a low, government-subsidized cost. In this way, hedge funds indirectly benefit from the implicit government subsidy protecting the banking system.

II. Empirical Motivation

In this section we present some empirical evidence that can be interpreted as support for the main implications of the model. While this paper is not intended as a primarily empirical study, our objective here is to motivate future empirical work on the relationship between banks and hedge funds. Beyond testing the model above, such future work also is important for policymakers and regulators considering the impact of hedge funds on systemic risk and financial market stability.

II.1 Hedge Fund Exposure to Systematic Liquidity Risk

The recent explosive growth of capital under hedge fund management raises the question of how much hedge fund capital is exposed to systematic liquidity shocks. The LTCM episode is the best known evidence of the massive proportions of this relationship. However, since liquidity events of such magnitude are relatively rare and reliable hedge fund data is available only for recent years,
the empirical answers of this question are only likely to emerge in the future.

What is a systematic liquidity shock? In this paper, we adopt a measure of "flight to quality" that has been shown to empirically capture liquidity inflows into the banking system: we follow Gatev and Strahan (2006) and use non-financial commercial-paper-Treasury-bill spread (paper-bill spread) as our measure of market liquidity. Our choice is primarily motivated by the fact that systematic inflows of bank liquidity are of key importance in our model above, because they help the bank estimate market-wide re-allocations that need not be related to idiosyncratic credit risk. This measure is imperfect, as it focuses on the commercial paper market, which may not have the highest exposure to systematic liquidity shocks, and some hedge funds may be in altogether uncorrelated markets. Nevertheless, to the extent that "flight to quality" is reflected in the changes of the paper-bill spread, hedge fund exposure to it can be interpreted as exposure to systematic liquidity shocks. First, we use this measure to assess the exposure of some hedge fund indexes.

Table I presents summary statistics for several publicly available hedge fund indexes that are compiled by Credit-Suisse/Tremont and are included in the TASS database.16 These indexes cover the main broad hedge fund styles and are constructed as monthly returns of portfolios of hedge funds that classify themselves according to a certain style. The hedge fund index data used here consist of monthly hedge fund index returns over the period 1994-2006.

Table I about here

Table II presents time series regression results for the Credit-Suisse/Tremont hedge fund indexes. The time series regressions include as explanatory variables five factors that capture the systematic components of a broad range of hedge funds: stocks (Wilshire 5000), bonds (Lehman Intermediate term corporate bond index), commodities (the Goldman Sachs Commodity Index) and currencies (the US Dollar Index). In addition, a fifth factor is included as proxy for liquidity shocks, that is constructed following Gatev and Strahan (2006), as the change in the spread between the 3-month commercial paper of large non-financial corporations and the 3-month Treasury bill rate.17

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16 See http://www.hedgeindex.com
17 All results reported below are robust to the inclusion of additional factors, like the Morgan-Stanley Emerging Index and changes in the implied volatility of index options, in an 8-factor regression specification. The results are very
We estimate the five factor loadings for each hedge fund style index using the time-series OLS regression,

\[
\text{Return}_t = \alpha + \beta_{Wish5k}r_{Wish5k,t} + \beta_{Bond}r_{Bond,t} + \beta_{GSCI}r_{GSCI,t} + \beta_{USDX}r_{USDX,t} \\
+ \beta_{DCPTB}\Delta(CP_{3\text{-month},t} - TBill_{3\text{-month},t}) + \epsilon_t
\]

(1)

The table includes a panel for the full sample, as well as a panel excluding the three months of August, September and October 1998, when a large systematic liquidity shock occurred, associated with the Russian default and the related subsequent collapse of LTCM.

The highlights of the table are the significant negative paper-bill spread exposures for several of the style indexes compiled by Credit-Suisse/Tremont. In particular, the table shows that portfolios of hedge funds that specialize in Convertible Arbitrage, Dedicated Short Bias, Event Driven Arbitrage, Distress Events and Event Driven Multi-Strategy have substantial exposure to liquidity shocks. The negative coefficients mean that the style portfolios on average loose value when the paper-bill spread widens. Moreover, removing the 1998 observations does not eliminate the results, except for the convertible arbitrage portfolio: while the coefficients drop by about 20%, they are still large and economically significant. Taken together, the results suggest that in aggregate, substantial amount of capital invested in hedge funds is exposed to short-term systematic liquidity shocks. Such transitory shocks potentially threaten hedge funds with “limited arbitrage” financial constraints arising from the hedge fund opaqueness.

The results highlight the fact that a well-diversified portfolio of loans to hedge funds may have substantial exposure to systematic liquidity shocks, which from a bank’s perspective is important for liquidity risk management. In particular, banks have to account for their total portfolio exposure to systematic liquidity risk and evaluate it relative to the their capacity to provide liquidity. The liquidity risk-bearing capacity of banks is a combination of the natural hedge from deposit inflows, determined by the size of the deposit franchise, and the availability of liquid asset reserves.

similar and are not reported here.
II.2 Hedge Fund Use of Backup Liquidity Lines

There is some existing empirical evidence that hedge funds use bank credit lines. Since hedge funds are not regulated and do not disclose details about their borrowing from banks, all the evidence to date has been found in data provided by banks. In particular, bank surveys conducted by European regulators show that banks extend credit lines to hedge funds.\textsuperscript{18}

III. The Model

III.1 Assumptions

Consider a hedge fund that borrows from a bank during a systematic liquidity shock. Both the bank and the hedge fund are assumed to maximize profit, subject to constraints.

At time 0, the hedge fund decides how much to borrow and how much backup liquidity to reserve from the bank. The information asymmetry is as follows. The hedge fund chooses its exposure to systematic liquidity shocks. This exposure is not known to outsiders, including the bank. This assumption captures the stylized fact that hedge funds have incentives to protect their proprietary investment strategies from being revealed. The bank has a prior on the expected return and variance of the hedge fund, but does not know the particular decomposition of the return into two components: systematic liquidity exposure and idiosyncratic management decision.

At time 1, a systematic liquidity shock has occurred and the hedge fund is assumed to have some leverage (which could be zero). There is one period remaining. We consider a bank’s decision to extent additional credit through the hedge fund’s backup liquidity credit line. The bank chooses the optimal amount to lend at time 0, by adjusting the hedge fund margin requirement. We assume that the pricing of the credit facilities is competitive. The hedge fund maximizes profit, choosing its optimal leverage and systematic liquidity risk exposure. The notation is as follows.

III.1.1 Bank Deposits and Opportunity Cost of Lending

Denote the bank’s transaction deposits at time 0 as \( D = D_0 + \Delta D \), where \( D_0 \) is the bank’s deposit base in place before time 0, that is, before the deposit flow \( \Delta D \) at time 0. The bank does not observe the liquidity shock \( S \) at time 0, but the deposits (in)flow, \( \Delta D \) is assumed correlated with

\textsuperscript{18}See ECB November 2005 report
In particular, we assume later that the inflow and the correlation are higher for larger transaction deposit franchise, $D_0$, in place at time 0 (see section II.4. below).

The size of the transaction deposit franchise, $D$ determines backup lending capacity at time 0 as follows. Deposit inflows do not increase the bank capital but increase the bank’s liquid assets. This lowers the opportunity cost of funding at time 0. Denote the bank’s opportunity cost of lending at time 0 as $Q(D) = Q_0$. We assume that $Q'(\cdot) < 0$. Intuitively, the bank opportunity set of loans with a given level of credit risk is limited and deposit inflows lower the funding cost of the bank.\(^{19}\)

If the bank chooses to hold illiquid assets with value $P_0$ that cannot be hedged, then it sets the risk premium for the illiquid asset risk in proportion to the opportunity cost of lending. We assume that the risk premium, denoted $\lambda$, is increasing as the bank’s opportunity cost of capital $Q_0$ increases. Denote $q_0 = \ln Q_0$ and specify a linear relationship writing,

$$\lambda = (a + bq_0)P_0$$  (2)

Let $i_0$ be the fixed bank interest rate on funds lent to the hedge fund prior to time 0. This interest is paid at time 0 and at time 1. Similarly, denote the fixed bank interest rate on funds lent at time 0 as $i_1$. This interest is paid at time 1.\(^{20}\) The banking sector is assumed competitive, so that the interest rates, $i_0$ and $i_1$ charged by the bank do not depend on the size of the bank’s deposit franchise.

**III.1.2 Hedge Fund Investment and Liquidity Risk Exposure**

At time 0, the hedge fund has capital $V_0$ and has borrowed $L_0$ from the bank. If at time 0 an additional amount $B_0$ is borrowed from the bank using a liquidity backup line, bringing total borrowing to $L'_0 = L_0 + B_0$, then time-0 investment, $I_0$, is financed in part by the borrowing $L'_0$, so that $I_0 = V_0 + L_0 + B_0$. The hedge fund invests $I_0$ in possibly illiquid shares and the price of a

\(^{19}\)Without the cash inflow, the funds would have to be raised at a higher cost. In particular, interest paid on transaction deposits is lower than LIBOR or the rate of Fed funds, which would reflect the external cost of liquidity if the banks needs it at time 0.

\(^{20}\)The fixed rates $i_0, i_1$ assumed here can be justified with competitive zero-profit short-term lending by large banks, with credit rationing (see Stiglitz and Weiss (1981)) of risky borrowers due to information asymmetry. Our focus in this paper is not the equilibrium which determines the competitive lending rates in the banking sector. See, for example, Boot, Thakor, and Udell (1987) and Thakor and Udell (1987) for analysis of the pricing structure of loan commitments as a way to separate high-quality from low-quality borrowers.
share is \( P_0 = 1 + R_0 \). The hedge fund’s investment return, \( R_1 \), is realized at time 1, bringing the NAV to \( V_1 = I_0(1 + R_1) - L_0(1 + i_0) - B_0(1 + i_1) \).

At time 0, a random event, \( S \), interpreted as short-term systematic liquidity shock has occurred, depressing the value of illiquid assets with positive exposure to the shock. The shock is not observable directly, but affects assets through their exposure to it. Denote \( -r_S = \ln(1 - R_S) \) where \( 1 - R_S \) is the market value loss, due to the liquidity shock, on 1 dollar invested in an asset with unit exposure to the shock.

The hedge fund investment in an illiquid asset, with exposure \( \gamma > 0 \) to the systematic liquidity shock, \( S \) has returns \( 1 + R_0 = (1 + R_{H,0})(1 - R_{L,0}) \) and \( 1 + R_1 = (1 + R_{H,1})(1 + R_{L,1}) \), with return component, \( R_L \), due to exposure to the systematic liquidity shock, as specified below. Denote, \( r_t = \ln(1 + R_t), r_{H,t} = \ln(1 + R_{H,t}), t = 1, 2, \) and \( -r_{L,1} = \ln(1 - R_{L,0}), r_{L,1} = \ln(1 + R_{L,1}) \).

If the illiquid asset has exposure \( \gamma \) to the liquidity shock \( S \), we have

\[
\begin{align*}
    r_{L,0} &= \gamma r_S \\
    r_{L,1} &= \gamma r_{S,1}
\end{align*}
\]

Assume that returns are log-normally distributed:

\[
\begin{align*}
    r_{H,1} &= \ln(1 + R_{H,1}) \sim N(\mu_A, \sigma_A^2) \\
    r_{S,1} &= \ln(1 + R_{S,1}) \sim N(r_S, \sigma_S^2)
\end{align*}
\]

so that

\[
    r_1 = r_{H,1} + \gamma r_{S,1} \sim N(\mu_A + \gamma r_S, \sigma_A^2 + \gamma^2 \sigma_S^2)
\]

The illiquid investment is more risky. If a transitory systematic liquidity shock realization, \( -r_S < 0 \) depresses the price of the illiquid asset with positive exposure to the shock, \( \gamma \), then this price shock is expected to revert with an expected reversal of the transitory systematic shock. Thus, if the hedge fund has invested in the illiquid asset with exposure, \( \gamma > 0 \), such liquidity shock improves the hedge fund opportunity, but the hedge fund cannot raise additional capital due to performance-
III.1.3 Transaction Deposits

Denote, $D = D_0(1 + R_D)$, $r_H = \ln(1 + R_{H,0})$, $r_S = -\ln(1 - R_S)$, $r_D = \ln(1 + R_D)$. The returns $1 + R_{H,0}, 1 - R_S, 1 + R_D$ are jointly-lognormally distributed:

$$
\begin{pmatrix}
  r_H \\
  r_S \\
  r_D 
\end{pmatrix}
\sim N
\begin{pmatrix}
  \mu_A \\
  0 \\
  \mu_D 
\end{pmatrix}
\begin{pmatrix}
  \sigma_A^2 & 0 & 0 \\
  0 & \sigma_S^2 & \sigma_{S,D} \\
  0 & \sigma_{S,D} & \sigma_D^2 
\end{pmatrix}
\tag{8}
$$

Based on the results of Gatev and Strahan (2006), assume that the correlation $\rho_{S,D}(D) = \frac{\sigma_{S,D}}{\sigma_S \sigma_D}$ is increasing in the size of the transaction deposits franchise, $D_0$. Next, motivated by Diamond and Dybvig (1983), assume also that the variance $\sigma_D$ is decreasing with $D_0$, due to better diversification of deposit flows. We assume that,

$$
\frac{d\rho_{S,D}}{dD_0} > 0 \quad (9)
$$

$$
\frac{d\sigma_D}{dD_0} < 0 \quad (10)
$$

III.1.4 Notation

It will be convenient to use the following notation. Using the labels $P_0 = 1 + R_0$, $P_1 = P_0(1 + R_1)$, introduced above, we also denote,

$$
n(B_0) = \frac{V_0 + L_0 + B_0}{P_0} \quad (11)
$$

$$
X(B_0) = \frac{L_0(1 + i_0) + B_0(1 + i_1)}{n} = \frac{L_0(1 + i_0) + B_0(1 + i_1)}{V_0 + L_0 + B_0} P_0 \quad (12)
$$

$$
Y_{HF}(B_0) = E_{HF}[\max(P_1 - X, 0)] \quad (13)
$$

$$
W_{BK}(B_0) = E_{BK}[\max(X - P_1, 0)] \quad (14)
$$

$$
\pi_{HF}(B_0) = nY_{HF} \quad (15)
$$

$$
y_t = \ln(1 + i_t), \; t = 0, 1 \quad (16)
$$

---

21Note that there is no flight of capital, as in Shleifer-Vishny (1997), because hedge funds use lock-up and advance-notice provisions.

22Diamond and Dybvig (1983) argue that by pooling their funds in a bank, agents can insure against idiosyncratic liquidity shocks and still invest most of their wealth in high-return illiquid projects.
and note that \( P_0/X > 1 \) if and only if \( V_0 > L_0i_0 + B_0i_1 \).

### III.2 The Hedge Fund Problem

The hedge fund borrows and invests the funds in a risky illiquid asset. Limited liability implies that the hedge fund position is equivalent to a call option. At time 0 the hedge fund problem is to choose the additional borrowing, \( B_0 \):

\[
\max_{B_0} E_{HF}[\max \left((V_0 + L_0 + B_0)(1 + R_1) - L_0(1 + i_0) - B_0(1 + i_1), 0\right)] \tag{17}
\]

s.t.

\[
\frac{1 - m}{m} V_0 - L_0 - B_0 \geq 0 \tag{18}
\]

\[
L_0(1 + i_0 - Q_0) + B_0(1 + i_1 - Q_0) - E_{BK}[\max(L_0(1 + i_0) + B_0(1 + i_1) - (V_0 + L_0 + B_0)(1 + R_1), 0)] \geq 0 \tag{19}
\]

where \( i_1 \) is the interest rate on the additional borrowing, \( B_0 \), and the subscripts \( HF \) and \( BK \) denote expectations conditional on the information available to the hedge fund and the bank respectively. The first constraint is a margin requirement (over-collateralization) on hedge fund borrowing with a margin \( m = \frac{V_0}{i_0} \), while the second bank participation constraint requires a non-negative net expected return for the bank lending to the hedge fund. This constraint is related to the bank problem and is discussed further below.

We can highlight the fact that the hedge fund is holding an option by writing the hedge fund problem (using the notation introduced above) as,

\[
\max_{B_0} \pi_{HF}(B_0) \tag{20}
\]

s.t.

\[
\frac{1 - m}{m} V_0 - L_0 \geq B_0 \tag{21}
\]

\[
n(X - W_{BK}) - Q_0(L_0 + B_0) \geq 0 \tag{22}
\]

The option on an illiquid asset, \( Y_{HF} \), (as well as the put option, \( W_{BK} \)) with risk-free rate, \( r_f = 0 \), \( T = 1 \), can be valued as follows. The bank sells the option that cannot be hedged continuously, and sets the risk premium for the illiquid asset risk. As stated above (equation (2), we assume the risk premium \( \lambda \) is increasing as the bank’s opportunity cost of capital \( Q_0 \) increases, \( q_0 = \ln Q_0 \), and
\( \lambda = (a + bq_0)P_0 \). Without loss of generality, we use \( a = 0, b = 1 \), so that,

\[ \lambda = q_0P_0 \quad (23) \]

Intuitively, higher opportunity cost implies higher required return per unit risk for bank funds lent, which implies higher price of risk. In this case, the equation for the call option price \( F \) is

\[
\frac{1}{2} \sigma^2 A P_0 + [\mu_A + \gamma r S - q_0] P_0 F_{F_0} + F_t = 0 \quad (24)
\]

with the usual put-option payoff boundary conditions. Intuitively, this equation reflects the condition that the "hedge" portfolio that is formed by the option and the underlying illiquid asset is not riskless, because a continuous hedge cannot be implemented through continuous trading of the illiquid asset. Consequently, the expected return on the hedge portfolio is not the risk-free rate, but the rate of return required by the bank as compensation for bearing the risk. The option value is given by,

\[
Y_{HF} = P_0 e^{\mu_A + \gamma r S - q_0} \Phi(d_1) - X \Phi(d_2) 
\]

\[
d_1 = \frac{\ln(P_0 e^{\mu_A + \gamma r S - q_0}/X) + \frac{\sigma^2}{2}}{\sigma},
\]

\[
d_2 = d_1 - \sigma = \frac{\ln(P_0 e^{\mu_A + \gamma r S - q_0}/X) - \frac{\sigma^2}{2}}{\sigma} \quad (26)
\]

where \( \Phi(.) \) denotes the cumulative standard normal distribution. This reduces to the standard Black-Scholes formula for liquid assets, \( \gamma = 0 \), as \( q_0 = \mu_A \).

Next, we can show (see the Appendix) that

\[
\frac{d\pi_{HF}}{dB_0} > 0 \text{ if and only if } e^{\mu_A + \gamma r S - q_0} > \Phi(d_2) \frac{\Phi(d_1)}{\Phi(d_2)} 
\]

and a sufficient condition for hedge fund profit to increase with increasing additional borrowing is, for example,

\[
\mu_A + \gamma r S > q_0 + y_1 
\]

Moreover, we consider below sufficient conditions that the bank participation constraint is satisfied, so that for a given \( i_1 \), a sufficiently large liquidity shock \( r_S > 0 \) implies that maximal borrowing,

\[
B_0^* = \frac{1-m}{m} V_0 - L_0 \quad (27)
\]

is optimal.\( ^{23} \) In fact, we will show that for low bank opportunity cost, for

\[
\text{This case is the first-best solution, where the bank and the hedge fund estimate the same. A second-best outcome due to binding bank participation constraint implies that the hedge fund optimally uses liquidity backup lines, as in Holmstrom and Tirole (2001).}
\]
example in the case of a large liquidity shock, and a bank with large transaction deposits franchise, the bank will lower the margin \( m \) at time 0, so that the hedge fund is able to borrow at least the full amount of the liquidity backup line commitment at the new lower margin. The above analysis can be summarized as,

Proposition 1. **Liquidity insurance is ex-post optimal for the hedge fund.** A sufficiently large systematic liquidity shock implies maximum hedge fund leverage.

Intuitively, if the temporarily depressed price of the illiquid asset is low enough relative to the cost of borrowing, the hedge fund wishes to invest more funds in the asset.

Consider next how hedge fund profit depends on the liquidity exposure \( \gamma \). Suppose that the hedge fund borrows the optimal amount, \( B_0^* > 0 \), which in the case of a large liquidity shock, \( r_S \) is the maximum amount lent by the bank at the given margin, \( m \). Denote, \( n^* = n(B_0^*), X^* = X(B_0^*), Y^*_HF = E_{HF}[\max (P_1 - X^*, 0)] \) and \( \pi^*_HF = n^*Y^*_HF \) and set \( P = P_0e^{\mu_A + \gamma r_S - q_0} \). We can show that,

\[
\frac{\partial \pi^*_HF}{\partial \gamma} = n^*P_0e^{\mu_A + \gamma r_S - q_0}[\Phi(d_1^*)r_S + e^{-\frac{d_1^2}{2}}\frac{\sigma^2 S^2 \gamma}{\sigma}] \tag{29}
\]

A sufficient condition for this expression to be positive is that the liquidity shock depresses the price of the illiquid asset: \( r_S > 0 \). Intuitively, more liquidity exposure increases the value of the hedge fund call option because it both increases the volatility and it makes the option appear deeper in-the-money. Therefore, in the absence of funding constraints, given a liquidity shock that depresses the price, it is ex-post optimal for the hedge fund to be invested in illiquid assets with the maximum exposure to systematic liquidity shocks.

Next, we can show that the hedge fund profit sensitivity to the magnitude of the liquidity shock is,

\[
\frac{\partial \pi^*_HF}{\partial r_S} = n^*\Phi(d_1^*)P_0e^{\mu_A + \gamma r_S - q_0} \gamma > 0 \tag{30}
\]

This result confirms our intuition that, the hedge fund profit increases with a larger liquidity shock that depresses the price. Increasing the magnitude of the liquidity shock implies that the expected return on the illiquid asset is higher, due to the expected higher reversion of the liquidity shock. This makes the hedge fund option more likely to expire in the money and hence it is more valuable.
Finally, consider how the expected profit depends on the size of the bank’s deposit base, \( D \). We can also show that, (see the appendix),

\[
\frac{\partial \pi^*_H}{\partial D} = -n^\ast \Phi(d_1^\ast)P_0e^{\mu A + \gamma S - \psi_0}\frac{1}{Q_0} \frac{dQ}{dD} > 0
\] (31)

This intuition of this result is that the hedge fund profit decreases when the bank opportunity cost of lending is increasing. Conversely, a higher deposit base implies a lower opportunity cost of lending and the hedge fund option is more valuable, because the bank requires a lower rate of return to compensate it for bearing liquidity risk. We can summarize this result as follows,

**Proposition 2. Funding cost advantage of a large transaction deposit franchise.** If the bank optimally chooses to lend to the hedge fund, then increasing the transaction deposit base of the bank implies increasing expected profit of the hedge fund.

This result implies that hedge funds are better off purchasing backup liquidity insurance from banks with larger transaction deposit franchises, provided that such banks are able to correctly identify the systematic liquidity shock as a market-wide event rather than a hedge fund-specific credit problem. We will consider below how the size of the deposit base affects the banks ability to estimate the magnitude of the systematic liquidity shock.

### III.3 The Bank Problem

Consider the bank problem at time 0. If the hedge fund performs well at time 1, then the bank receives principal plus interest owed, otherwise it recovers a lower amount. The limited maximum bank cashflows possible imply that at time 0, the bank effectively sells the hedge fund a put option on the illiquid asset held by the hedge fund. Denote \( I_1 = I_0(1 + R_1) = (V_0 + L_0 + B_0)(1 + R_1) \).

The bank’s net (total) return is

\[
L_0(1 + i_0 - Q_0) + B_0^\ast (1 + i_1 - Q_0) - \max[L_0(1 + i_0) + B_0^* (1 + i_1) - I_1, 0]
\] (32)

where \( B_0^\ast \) is the optimal additional borrowing for the hedge fund. Use the notation \( n^\ast, X^\ast \) as before, and label the put option value as,

\[
W_{BK} = E_{BK}[\max(X^\ast - P_1, 0)]
\] (33)
This option on the illiquid hedge fund asset has value,

\[
W_{BK} = E_{BK}[\max(X^* - P_1, 0)] = X^*\Phi(-d_2^*) - P_0e^{\mu_A + \gamma r_S - q_0}\Phi(-d_1^*)
\]  
(34)

\[
d_1^* = \frac{\ln(P_0e^{\mu_A + \gamma r_S - q_0}/X^*) + \sigma^2}{\sigma}, \quad d_2^* = d_1^* - \sigma = \frac{\ln(P_0e^{\mu_A + \gamma r_S - q_0}/X^*) - \sigma^2}{\sigma}
\]  
(35)

The bank net expected return is:

\[
\pi_{BK} = n^*(X^* - W_{BK}) - Q_0(L_0 + B_0^*)
\]  
(36)

The bank maximizes net expected return, choosing the margin \( m \) on time-0 lending

\[
\max_{m \geq 0} E_{BK}[\pi_{BK}]
\]  
(37)

Suppose it is optimal for the hedge fund to borrow the maximum amount available, as in the case of a sufficiently large liquidity shock \( r_S > 0 \) discussed above. In this case we can show that (see Appendix),

\[
\frac{\partial \pi_{BK}}{\partial m} = -\frac{V_0}{m^2}[e^{y_1}\Phi(d_2^*) + e^{\mu_A + \gamma r_S - q_0}\Phi(-d_1^*) - e^{q_0}]
\]  
(38)

and if \( q_0 < y_1 \), that is, when the bank opportunity cost of lending is lower than the competitive lending rate, we have

\[
\frac{\partial \pi_{BK}}{\partial m} < -\frac{V_0}{m^2}[e^{\mu_A + \gamma r_S - q_0}\Phi(-d_1^*) - e^{y_1}\Phi(-d_2^*)]
\]  
(39)

A sufficient condition for the right-hand side to be negative is, for example (see the Appendix),

\[
\mu_A + \gamma r_S > q_0 + y_1 + \ln 2
\]  
(40)

which would be the case if there is a large liquidity shock, \( r_S > 0 \) that depresses the price of the illiquid asset and lowers the opportunity cost of the bank. The intuition of this result is that the high expected return of the hedge fund, that is due to the expected reversal of the systematic liquidity shock, makes the put option sold by the bank less valuable: the high expected return is similar to a "negative dividend", that increases the price of the option when it is paid out. In such case, the bank has incentive to allow maximum hedge fund borrowing on the backup line by lowering the margin if necessary, so the bank and hedge fund incentives are aligned.

Otherwise, if \( q_0 > y_1 \), we can show that (see the Appendix),

\[
\frac{\partial \pi_{BK}}{\partial m} > -\frac{V_0}{m^2}[e^{\mu_A + \gamma r_S - q_0}\Phi(-d_1^*) - e^{y_1}\Phi(-d_2^*)]
\]  
(41)
A sufficient condition for the right-hand side to be positive is,

\[ \mu_A + \gamma r_S < q_0 + y_1 \]  \hspace{1cm} (42)

for example, if there is a "reverse" liquidity shock, \( r_S < 0 \), that increases the price of the illiquid asset and the opportunity cost of the bank. We can summarize this analysis in the following,

**Proposition 3.** Backup liquidity lending is ex-post optimal for bank. If there is a sufficiently large liquidity shock that depresses the price, it is ex-post optimal to lend more to the hedge fund.

Combined with proposition 1, this result shows that liquidity backup lines that are used during systematic liquidity shocks enable equilibrium risk sharing between banks and hedge funds.

Consider now the dependence of bank profit on liquidity exposure \( \gamma \). We can show that (see the Appendix),

\[
\frac{\partial \pi_{BK}}{\partial \gamma} = n^*P_0e^{\mu_A + \gamma r_S - q_0}[\Phi(-d_1)r_S - e^{-\frac{d_1^2}{2}}\sigma^2S\gamma] \]  \hspace{1cm} (43)

This expression is positive for the limit \( \gamma = 0 \), as long as \( r_S > 0 \), so by continuity, it is positive for small \( \gamma \), which means that increasing the liquidity risk exposure above zero makes the bank better-off. On the other hand, for sufficiently high liquidity exposure, the bank profit may decrease as the exposure is increased further. Intuitively, the tradeoff is that more exposure increases the volatility and makes the option that the bank sold more valuable, but more exposure also makes the option more out-of-the-money, because of the higher drift.

Next, we can show that the bank net expected return sensitivity to the magnitude of the liquidity shock is (see the Appendix),

\[
\frac{\partial \pi_{BK}}{\partial r_S} = n^*\Phi(-d_1)P_0e^{\mu_A + \gamma r_S - q_0}\gamma > 0 \]  \hspace{1cm} (44)

Intuitively, the bank is better off when the realized liquidity shock depressing the price is larger, because that makes the put option written by the bank appear deeper out-of-the-money. We now consider the bank’s ability to estimate systematic liquidity shocks.

**III.4 The Information Role of Transaction Deposits**

Consider now the bank estimates the realization of the systematic liquidity shock, \( r_S \). The bank...
does not observe the liquidity shock realization, \( r_S \), but observes its transaction deposits inflow, \( r_D \), which is normally distributed and so the conditional distribution of \( r_S \) given \( r_D \) is known,

\[
(r_S|r_D) \sim N\left(\frac{\sigma_{S,D}}{\sigma_D^2}(r_D - \mu_D), \sigma_S^2 - \frac{\sigma_{S,D}^2}{\sigma_D^2}\right)
\]

The conditional mean estimate, \( \hat{r}_S = E[r_S|r_D] \), has the following properties:

\[
\hat{r}_S = \frac{\sigma_{S,D}}{\sigma_D}(r_D - \mu_D)
\]

\[
d\hat{r}_S = \frac{\sigma_S}{\sigma_D^2}(r_D - \mu_D)\left[\frac{d\rho_{S,D}}{dD_0}\sigma_D - \rho_{S,D}\frac{d\sigma_D}{dD_0}\right]
\]

Given the correlation structure above, this expression is positive, as long as \( r_D - \mu_D > 0 \). Intuitively, given a positive abnormal transaction deposits inflow realization, which would be the typical flight to quality scenario, \( r_D - \mu_D > 0 \) and a large transaction deposits franchise, \( D_0 \), implies a higher estimate of \( -\hat{r}_S \), because a correlated deposit inflow is more likely, \( \rho_{S,D} \) is higher, and the variation of the inflow, \( \sigma_D \) is lower, due to better diversification.

Suppose a hedge fund assesses whether the bank is likely to lend in the case of a liquidity shock \( r_S > 0 \) that depresses the price of the illiquid asset. The hedge fund considers the bank’s expected inflow in the case of the liquidity shock, which is conditionally normally distributed with mean,

\[
E[r_D|r_S] = \mu_D + \frac{\sigma_{S,D}}{\sigma_D^2}r_S
\]

The expected inflow is larger for banks with larger transaction deposits and increases with a larger liquidity shock. With this expected inflow, the hedge fund expects the bank’s estimate of the liquidity shock to be,

\[
E_{HF}[\hat{r}_S] = \frac{\sigma_{S,D}^2}{\sigma_D^2}r_S
\]

\[
dE_{HF}[\hat{r}_S] = 2r_S\frac{\sigma_{S,D}\sigma_S}{\sigma_D^4}\left[\frac{d\rho_{S,D}}{dD_0}\sigma_D - \rho_{S,D}\frac{d\sigma_D}{dD_0}\right] > 0
\]

so that a bank with a larger transaction deposit franchise would estimate a higher shock. Intuitively, with a higher size of transaction deposits franchise, \( D_0 \), the bank is expected to receive a higher inflow into transaction deposits, because the inflow is more correlated with the liquidity shock. The higher inflow in turn implies a higher bank estimate for the liquidity shock realization.

We can summarize this analysis in the following,
Proposition 4. Information advantage of a large transaction deposit franchise. If the bank experiences a positive abnormal transaction deposit inflow, then a higher transaction deposit base implies a higher estimate of the liquidity shock realization, with lower error.

Combined with proposition 3, this result implies that hedge funds are better off purchasing backup liquidity insurance from banks with larger transaction deposit franchises. Such banks would have a better estimate of the systematic liquidity shock realization and, as a result, would be more likely to lend to the hedge fund during such tight liquidity events. This implication highlights the additional informational role of transaction deposits that gives the bank a unique advantage in lending to hedge funds.

III.5 Empirical Implications

The model presented above shows that transaction deposit inflows can provide additional information that helps identify a systematic liquidity shock. One important implication of the analysis is that the size of the deposit franchise of a particular bank determines its hedging capacity through the size of the inflows that the bank receives during a systematic liquidity shock. Moreover, a bigger franchise improves the precision of the signal about short-term flights to quality that are not related to hedge fund specific credit risks.

These results have cross-sectional implications about the backup liquidity commitments extended to hedge funds by different banks. The prediction is that since banks with more transaction deposits have more hedging capacity and get a more precise signal about liquidity shocks, they optimally would make more backup liquidity available to hedge funds. Recent survey data suggests that hedge funds spend substantial amounts on prime brokerage, including backup liquidity lines.24 The main prime brokers are large investment banks who purchase liquidity insurance from large commercial banks that have substantial transaction deposit franchises. This pairing is consistent with the implications of the analysis above.

Another empirical implication is that banks with larger transaction deposits lend more to hedge funds during systematic liquidity shocks, possibly lowering margin requirements. The current lack

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of public bank disclosure on business with hedge funds precludes direct tests of these empirical implications by academicians. The data requirements include the amount of backup lines extended to hedge funds. Such data are available internally to bankers and presumably to their FDIC examiners, but at the time of this writing they are kept confidential.

These cross-sectional implications would also be useful to regulators aiming to assess the exposure of banks to hedge funds. Interpreted in this way, it offers a diagnostic that may reveal potential unhedged liquidity risk exposures for banks that extend a significant amount of backup liquidity lines without the transaction deposit franchise, to offset the aggregate exposure from these lines.

The model also has cross-sectional implications for opportunistic hedge funds who avail themselves of bank credit lines. Such hedge funds would optimally choose to buy liquidity insurance from banks with large transaction deposit franchises, because the higher quality of the banks’ signal about the liquidity shock implies that they are more likely to facilitate borrowing of “bridge liquidity” through lowering of margins during times of stress. Moreover, the model predicts that opportunistic hedge funds with backup liquidity lines from banks with large transaction deposit franchises on average outperform peers who are unable to avail themselves of the lower-quality liquidity insurance offered by banks with less transaction deposits. Direct tests of these implications are difficult at the time of writing, due to lack of hedge fund disclosure on the use of bank credit lines.

IV. Welfare Analysis: An Illustration

The results developed so far show that during large systematic liquidity shocks, hedge funds optimally increase leverage to the maximum feasible level. This borrowing is possible because of the optimality of increased bank lending, when bank deposits experience large inflows during systematic liquidity shocks. These bank inflows materialize because of the government safety net that protects deposits. In this section, we consider the welfare implications of this arrangement in a stylized setting that highlights the importance of the government safety net.

The optimality of the government safety net, in its current form, depends on the implied costs and benefits for different agents. At present, our understanding of the allocation and properties of these costs and benefits is rather limited. Hence, our objective in this section is to highlight
the importance of future research in this direction, rather than to provide a welfare analysis in full generality. In particular, we develop the simple intuition that if the advent of hedge funds has changed the allocations of costs and benefits for different types of agents, such as hedge fund operators and non-operators, then the particular structure of the government safety net of the past may no longer be optimal, in the sense of maximizing social surplus. For concreteness, the following analysis is carried out under some simple assumptions about the government safety net and its costs and benefits, fully recognizing that future empirical analysis may come to different conclusions about the properties and allocations of these costs and benefits. Hence, the results in this section are intended as an illustration of the general principle, and not as definitive conclusions. It is our hope that better informed future research will address these important questions in their full generality.

We treat the government safety net as a public good. For simplicity, suppose that the perceived safety of bank deposits is entirely the result of FDIC insurance.\textsuperscript{25} Denote by $z$ the dollar level of per-capita FDIC insurance, for example $100,000$. The cost of supplying $z$ dollars of per-capita deposit insurance is $c(z)$, where the function $c$ is twice differentiable and $c'(.) > 0$, $c''(.) > 0$.\textsuperscript{26} Suppose there are $N$ consumers with derived utility $\phi_n(z)$, $n = 1, ..N$ over the level $z$ of per-capita deposit insurance, and this function is twice differentiable, with $\phi'_n(.) > 0$, $\phi''_n(.) < 0$. Intuitively, for each consumer, the marginal benefit of an extra insured dollar is decreasing as the consumer deposits more, because the likelihood of an additional dollar deposited by the consumer is decreasing. The next assumption captures the intuition that the government safety net, modeled as the level of per-capita deposit insurance, is the outcome of optimal social policy.

**Assumption A:** The level of per-capita deposit insurance in place, $z^*$ maximizes the aggregate surplus in the absence of hedge funds.

$$z^* = \arg \max_{z \geq 0} \sum_{n=1}^{N} \phi_n(z) - c(z)$$ \hspace{1cm} (51)

\textsuperscript{25}This simplification allows a straightforward quantification of the output of public good. On the other hand, the quantification of implicit government guarantees is not trivial and it is not considered here, because it does not add any additional insight to the argument.

\textsuperscript{26}The increasing marginal cost is a standard assumption that can be justified by the increasing incentive distortions for the banks, as well as increasing coordination problems for the government.
This implies Pareto optimality of the government safety, as characterized by the deposit insurance level \(z^*\), in the absence of hedge fund borrowing. We now consider how this Pareto optimality is affected when we introduced hedge funds that benefit from bank lending during systematic liquidity shocks. The following assumption is motivated by the finding of Gatev and Strahan (2006):

**Assumption B**: The size of the banks’ transaction deposit franchise and deposit inflows during systematic liquidity shocks are increasing with higher level of per-capita deposit insurance.

\[
D_0 = k_0 z, \quad k_0 > 0 \quad (52)
\]
\[
1 + r_D = K z, \quad K > 0 \quad (53)
\]

Intuitively, a low level of per-capita deposit insurance \(z\) would attract smaller deposits on average, while simultaneously leaving less slack insured deposit capacity, to accommodate deposit inflows during systematic liquidity shocks.

Under this assumption, hedge funds managers, \(m = 1,..M \leq N\) have additional derived utility \(\psi_n(z), m = 1,..M\) over the level \(z\) of per-capita deposit insurance, related to their compensation as hedge fund managers. To see this, we note that hedge fund manager compensation is an increasing convex function of hedge fund value which, according to proposition 3, is increasing with increasing bank lending \(D_0\). As before, suppose that this function is twice differentiable, with \(\psi_n'(.) > 0, \psi_n''(.) > 0\).

When some consumers of deposit insurance are hedge fund managers with the above modified derived utility, the optimal level of deposit insurance that maximizes aggregate surplus, solves the problem

\[
z^* = \arg \max_{z \geq 0} \sum_{n=1}^{N} \phi_n(z) + \sum_{m=1}^{M} \psi_m(z) - c(z) \quad (54)
\]

Comparing this equation to the one in "Assumption A" above, it can be seen that the optimal level of deposit insurance is different for an economy with hedge funds that benefit from bank lending during systematic liquidity shocks. In fact, under these stylized assumptions, we can prove the following proposition (see the Appendix):

*Proposition 5. Optimality of higher level of deposit insurance in the presence of hedge funds.*
an economy with hedge funds, the level of per capital deposit insurance that maximizes aggregate surplus is higher than the optimal level in the absence of hedge funds.

Note that while in this case the introduction of hedge funds leads to a Pareto improvement, the safety net is not at its optimal level, in the sense of maximizing social surplus. The intuition of the proposition is that with the introduction of hedge funds, the sum of marginal benefits is higher with the extra derived utilities of the hedge fund managers, while the cost function does not change. The assumption that the addition of hedge funds does not change the cost function can be justified by appealing to the self-fulfilling expectations of the public that the safety net always prevents systemic failure: if sufficient funds enter the banking system during systematic liquidity shortages, because of these expectations, then the banking system will not fail due to lack of liquidity, and the safety net will not be used. On the other hand, if the substantial increase in hedge fund activity increases the likelihood of systemic failure and losses that are absorbed by the public, then the cost function is likely to be different. This may well be the case if deposit inflows become unable to offset the increasing demand for liquidity by a growing number of hedge funds. Future empirical analysis of these risks would help move us toward better understanding of the role and optimal format of the government safety net.

While the proposition and its implications depend on the particular assumptions about the mathematical behavior of the marginal cost and benefit of deposit insurance, in general, introducing hedge funds that benefit from government-protected deposit changes the optimal level of deposit insurance. In the particular scenario considered above, if the cost of supplying the public good is allocated among consumers so that each consumer bears an increasing fraction of it, then a sub-optimally high level of deposit insurance increases the benefit-to-cost ratio for hedge funds managers, while it decreases the ratio for non-hedge fund managers. Intuitively, with a higher cost of deposit insurance, each consumers bears a higher cost, but the corresponding increase in benefit applies only to the hedge fund managers. In this sense, if taxpayers ultimately finance the government safety net then they "pay" for the corresponding increase of wealth of hedge fund managers.
V. Conclusion

This paper develops a model of systematic liquidity risk-sharing between hedge funds and banks. The model highlights the importance of transaction deposits, first, as the source of cheap funding for banks and hence for hedge funds, and second as a source of additional information that banks use during market-wide transitory liquidity shocks.

The main conclusion from the paper is that hedge funds benefit substantially from the banks’ exclusive capacity for hedging systematic liquidity risk. Banks have exclusive access to cheap deposit funding that provides a natural hedge for systematic liquidity shocks and a private signal about such shocks. The special position of banks stems from the FDIC insurance and the implicit government safety net protecting bank deposits. As a result, hedge funds are able to purchase liquidity insurance from banks and boost their profits while simultaneously enhancing banks’ interest income and trading revenues.
References


European Central Bank, 2005, "Large EU Banks' Exposures to Hedge Funds”, November.


Studies.


Appendix

A.1. Hedge Fund Sensitivities

We use the notation introduced in the main exposition. Consider first the derivative of hedge fund profit, \( \pi_{HF} \) with respect to additional borrowing, \( B_0 \).

\[
\frac{dP_1}{dB_0} = 0 \quad (55)
\]

\[
\frac{dX}{dB_0} = \frac{[(V_0 + L_0 + B_0)(1 + i_1) - L_0(1 + i_0) - B_0(1 + i_0)](1 + R_1)}{(V_0 + L_0 + B_0)^2} \quad (56)
\]

\[
= \frac{V_0(1 + i_1) + L_0(i_1 - i_0)}{(V_0 + L_0 + B_0)^2}P_0 \quad (57)
\]

\[
\frac{dY_{HF}}{dB_0} = \frac{\partial Y_{HF}}{\partial X} \frac{dX}{dB_0} = -\Phi(d_2) \frac{V_0(1 + i_1) + L_0(i_1 - i_0)}{(V_0 + L_0 + B_0)^2}P_0 \quad (58)
\]

and

\[
\frac{dn}{dB_0} = \frac{1}{P_0} \quad (59)
\]

\[
\frac{d\pi_{HF}}{dB_0} = \frac{dn}{dB_0} Y_{HF} + n \frac{dY_{HF}}{dB_0} = \quad (60)
\]

\[
= e^{\mu_A + \gamma r_S - q_0} \Phi(d_1) - e^{y_1} \Phi(d_2) \quad (61)
\]

\[
\frac{d\pi_{HF}}{dB_0} > 0 \text{ if and only if } e^{\mu_A + \gamma r_S - q_0} \Phi(d_1) - e^{y_1} \Phi(d_2) \geq 1 \quad (62)
\]

and a positive argument of the exponent on the left hand side is a sufficient condition for positive derivative:

\[
\mu_A + \gamma r_S > q_0 + y_1 \quad (63)
\]

Next, consider the derivative of hedge fund profit with respect to systematic risk exposure \( \gamma \). Denote

\[
\sigma = \sigma_A \sqrt{1 + \gamma^2 \frac{\sigma^2_S}{\sigma_A^2}} \quad (64)
\]

We will use the following results.

\[
\frac{\partial \sigma}{\partial \gamma} = \frac{\sigma^2_S \gamma}{\sqrt{\sigma^2_A + \gamma^2 \sigma^2_S}} = \frac{\sigma^2_S \gamma}{\sigma} = \frac{\sigma_S}{\sqrt{1 + \frac{1}{\gamma^2} \frac{\sigma^2_A}{\sigma^2_S}}} \quad (65)
\]

\[
\frac{\partial d_1^*}{\partial \gamma} = \frac{\partial}{\partial \gamma} \left( \ln \left( \frac{P_1}{X_1^*} \right) + \frac{\mu_A + \gamma r_S - q_1}{\sigma} + \frac{\sigma}{2} \right) \quad (66)
\]
Next, the hedge fund profit sensitivity to the magnitude of the liquidity shock is,

\[ \frac{\partial \pi^*_H}{\partial \gamma} = \frac{r_s \sigma}{\sigma^2} \left( \ln \left( \frac{P_1}{X^*} \right) + \mu_A + \gamma r_S - q_1 \right) - \frac{\sigma^2 \gamma}{2} = \]  

\[ \frac{r_s \sigma}{\sigma^2} \left( \ln \left( \frac{P_1}{X^*} \right) + \mu_A + \gamma r_S - q_1 \right) - \frac{\sigma^2 \gamma}{2} = \]  

\[ \frac{r_s \sigma^2 \gamma}{\sigma} \left( \ln \left( \frac{P_1}{X^*} \right) + \mu_A + \gamma r_S - q_1 \right) - \frac{\sigma^2 \gamma}{2} = \]  

\[ \frac{r_s \sigma^2 \gamma}{\sigma^2} \left( \ln \left( \frac{P_1}{X^*} \right) + \mu_A + \gamma r_S - q_1 \right) - \frac{\sigma^2 \gamma}{2} = \]  

\[ \frac{r_s \sigma^2 \gamma}{\sigma^2} \left( \ln \left( \frac{P_1}{X^*} \right) + \mu_A + \gamma r_S - q_1 \right) - \frac{\sigma^2 \gamma}{2} = \]  

\[ \frac{r_s \sigma^2 \gamma}{\sigma^2} \left( \ln \left( \frac{P_1}{X^*} \right) + \mu_A + \gamma r_S - q_1 \right) - \frac{\sigma^2 \gamma}{2} = \]  

\[ \frac{r_s \sigma^2 \gamma}{\sigma^2} \left( \ln \left( \frac{P_1}{X^*} \right) + \mu_A + \gamma r_S - q_1 \right) - \frac{\sigma^2 \gamma}{2} = \]  

\[ \frac{r_s \sigma^2 \gamma}{\sigma^2} \left( \ln \left( \frac{P_1}{X^*} \right) + \mu_A + \gamma r_S - q_1 \right) - \frac{\sigma^2 \gamma}{2} = \]  

\[ \frac{r_s \sigma^2 \gamma}{\sigma^2} \left( \ln \left( \frac{P_1}{X^*} \right) + \mu_A + \gamma r_S - q_1 \right) - \frac{\sigma^2 \gamma}{2} = \]  

\[ \frac{r_s \sigma^2 \gamma}{\sigma^2} \left( \ln \left( \frac{P_1}{X^*} \right) + \mu_A + \gamma r_S - q_1 \right) - \frac{\sigma^2 \gamma}{2} = \]  

Differentiating profit with respect to systematic risk exposure \( \gamma \), we have,

\[ \frac{\partial \pi^*_H}{\partial \gamma} = r_s \sigma - \frac{\ln \left( \frac{P_1}{X^*} \right) + \mu_A + \gamma r_S - q_1}{\sigma} + \frac{\sigma^2 \gamma}{2} = \]  

\[ \frac{\partial \pi^*_H}{\partial \gamma} = r_s \sigma - \frac{\ln \left( \frac{P_1}{X^*} \right) + \mu_A + \gamma r_S - q_1}{\sigma} + \frac{\sigma^2 \gamma}{2} = \]  

\[ \frac{\partial \pi^*_H}{\partial \gamma} = r_s \sigma - \frac{\ln \left( \frac{P_1}{X^*} \right) + \mu_A + \gamma r_S - q_1}{\sigma} + \frac{\sigma^2 \gamma}{2} = \]  

\[ \frac{\partial \pi^*_H}{\partial \gamma} = r_s \sigma - \frac{\ln \left( \frac{P_1}{X^*} \right) + \mu_A + \gamma r_S - q_1}{\sigma} + \frac{\sigma^2 \gamma}{2} = \]  

\[ \frac{\partial \pi^*_H}{\partial \gamma} = r_s \sigma - \frac{\ln \left( \frac{P_1}{X^*} \right) + \mu_A + \gamma r_S - q_1}{\sigma} + \frac{\sigma^2 \gamma}{2} = \]  

\[ \frac{\partial \pi^*_H}{\partial \gamma} = r_s \sigma - \frac{\ln \left( \frac{P_1}{X^*} \right) + \mu_A + \gamma r_S - q_1}{\sigma} + \frac{\sigma^2 \gamma}{2} = \]  

A sufficient condition for this expression to be positive is that the liquidity shock depresses the price of the illiquid asset: \( r_S > 0 \).

Next, the hedge fund profit sensitivity to the magnitude of the liquidity shock is,

\[ \frac{\partial \pi^*_H}{\partial r_S} = n^* \frac{\partial Y^*_H}{\partial r_S} \frac{\partial P_1}{\partial r_S} = \]  

\[ \frac{\partial \pi^*_H}{\partial r_S} = n^* \frac{\partial Y^*_H}{\partial r_S} \frac{\partial P_1}{\partial r_S} = \]  

\[ \frac{\partial \pi^*_H}{\partial r_S} = n^* \frac{\partial Y^*_H}{\partial r_S} \frac{\partial P_1}{\partial r_S} = \]  

\[ \frac{\partial \pi^*_H}{\partial r_S} = n^* \frac{\partial Y^*_H}{\partial r_S} \frac{\partial P_1}{\partial r_S} = \]  

\[ \frac{\partial \pi^*_H}{\partial r_S} = n^* \frac{\partial Y^*_H}{\partial r_S} \frac{\partial P_1}{\partial r_S} = \]  

Finally, consider how the expected profit depends on the size of the bank’s deposit base, \( D \). We have,

\[ \frac{\partial \pi^*_H}{\partial D} = \frac{\partial \pi^*_H}{\partial q_0} \frac{\partial q_0}{\partial D} = n^* \frac{\partial Y^*_H}{\partial q_0} \frac{\partial P_1}{\partial q_0} Q_0 \frac{dQ}{dD} = \]  

\[ \frac{\partial \pi^*_H}{\partial D} = \frac{\partial \pi^*_H}{\partial q_0} \frac{\partial q_0}{\partial D} = n^* \frac{\partial Y^*_H}{\partial q_0} \frac{\partial P_1}{\partial q_0} Q_0 \frac{dQ}{dD} = \]  

33
\[ \frac{\partial \pi_{BK}}{\partial m} = -n^* \Phi(d_1^*) P_0 e^{\mu_A + \gamma r S - q_0} \int_0^{1/Q_0} \frac{dQ}{dD} > 0 \] (83)

The inequality follows from our assumption that \( \frac{dQ}{dD} < 0 \).

### A.2. Bank Sensitivities

Consider the sensitivity to the margin \( m \). We have,

\[ \frac{\partial \pi_{BK}}{\partial m} = \frac{dn^*}{dm} (X^* - W_{BK}) + n^* \frac{d(X^* - W_{BK})}{dm} - Q_0 \frac{dB^*_0}{dm} = \] (84)

\[ = -\frac{V_0}{P_0} \left( P_0 \frac{L_0(i_0 - i_1)m + (1 - m) V_0(1 + i_1)}{V_0} \Phi(d_2^*) + P_0 e^{\mu_A + \gamma r S - q_0} \Phi(-d_1^*) \right) \] (85)

\[ + \frac{1}{m^2} \Phi(d_2^*) (L_0(i_0 - i_1)m - V_0(1 + i_1)m) + \frac{Q_0}{m^2} V_0 = \] (86)

\[ = -\frac{V_0}{m^2} \left( \frac{L_0}{V_0} (i_0 - i_1)m + (1 - m)(1 + i_1) \Phi(d_2^*) + P_0 e^{\mu_A + \gamma r S - q_0} \Phi(-d_1^*) \right) + \] (87)

\[ + \frac{V_0}{m^2} \Phi(d_2^*) \left( \frac{L_0}{V_0} (i_0 - i_1)m - (1 + i_1)m \right) + \frac{V_0}{m^2} Q_0 = \] (88)

\[ = -\frac{V_0}{m^2} \left[ (1 + i_1) \Phi(d_2^*) + e^{\mu_A + \gamma r S - q_0} \Phi(-d_1^*) - Q_0 \right] \] (89)

\[ = -\frac{V_0}{m^2} \left[ e^{y_1} \Phi(d_2^*) + e^{\mu_A + \gamma r S - q_0} \Phi(-d_1^*) - e^{y_0} \right] \] (90)

and if \( q_0 < y_1 \), we have

\[ \frac{\partial \pi_{BK}}{\partial m} < -\frac{V_0}{m^2} \left[ e^{y_1} \Phi(d_2^*) + e^{\mu_A + \gamma r S - q_0} \Phi(-d_1^*) - e^{y_1} \right] = \] (91)

\[ -\frac{V_0}{m^2} \left[ e^{\mu_A + \gamma r S - q_0} \Phi(-d_1^*) - e^{y_1} \Phi(-d_2^*) \right] \] (92)

A sufficient condition for the right-hand side to be negative is,

\[ e^{\mu_A + \gamma r S - q_0 - y_1} > \frac{\Phi(-d_2^*)}{\Phi(-d_1^*)} = 1 + \int_{-d_1^*}^{-d_2^*} e^{-x^2} dx \] (93)

for example, since the integral is bounded by 1, a sufficient condition is

\[ \mu_A + \gamma r S > q_0 + y_1 + \ln 2 \] (94)

In the other case, if \( q_0 > y_1 \), a similar argument shows that,

\[ \frac{\partial \pi_{BK}}{\partial m} > -\frac{V_0}{m^2} \left[ e^{y_1} \Phi(d_2^*) + e^{\mu_A + \gamma r S - q_0} \Phi(-d_1^*) - e^{y_1} \right] = \] (95)

\[ -\frac{V_0}{m^2} \left[ e^{\mu_A + \gamma r S - q_0} \Phi(-d_1^*) - e^{y_1} \Phi(-d_2^*) \right] \] (96)
This last term is positive as long as,

\[ e^{\mu_A + \gamma r_S - q_0 - y_1} < \frac{\Phi(-d_2^*)}{\Phi(-d_1^*)} \] (97)

Hence, a negative argument of the exponent is a sufficient condition for the right-hand side to be positive,

\[ \mu_A + \gamma r_S < q_0 + y_1 \] (98)

for example, if \( \gamma > 0 \) and \( r_S < 0 \) has sufficiently large absolute value.

Next, consider the derivative of bank profit, \( \pi_{BK} \), with respect to hedge fund systematic risk exposure, \( \gamma \).

\[
\frac{\partial \pi_{BK}}{\partial \gamma} = -n^* \frac{\partial W_{BK}}{\partial \gamma} = -n^* \frac{\partial}{\partial \gamma} \left[ X^* \Phi(-d_2^*) - P_0 e^{\mu_A + \gamma r_S - q_0} \Phi(-d_1^*) \right] = \]

\[
= -n^* \left( X^* \frac{\partial \Phi(-d_2^*)}{\partial d_2^*} \frac{\partial d_2^*}{\partial \gamma} - P_0 e^{\mu_A + \gamma r_S - q_0} \left[ \frac{\partial \Phi(-d_1^*)}{\partial d_1^*} \frac{\partial d_1^*}{\partial \gamma} - \frac{\partial \Phi(-d_1^*)}{\partial d_1^*} \frac{\partial d_1^*}{\partial \gamma} \right] \right) = \]

\[
= -n^* \left( X^* \frac{\partial \Phi(-d_2^*)}{\partial d_2^*} \frac{\partial d_2^*}{\partial \gamma} - P_0 e^{\mu_A + \gamma r_S - q_0} \left[ \frac{\partial \Phi(-d_1^*)}{\partial d_1^*} - \frac{\sigma^2 \gamma}{\sigma^2} d_1^* \right] \right) = \]

\[
= -n^* \left( X^* \frac{\partial \Phi(-d_2^*)}{\partial d_2^*} \frac{\partial d_2^*}{\partial \gamma} - P_0 e^{\mu_A + \gamma r_S - q_0} \left[ \frac{\partial \Phi(-d_1^*)}{\partial d_1^*} - \frac{\sigma^2 \gamma}{\sigma^2} d_1^* + \frac{\sigma^2 \gamma}{\sigma} \right] \right) = \]

\[
= n^* P_0 e^{\mu_A + \gamma r_S - q_0} \left[ \Phi(-d_1^*) r_S - e^{-\frac{d_1^*}{2}} \frac{\sigma^2 \gamma}{\sigma} \right] \]

This expression is positive for the limit \( \gamma = 0 \), as long as \( r_S > 0 \), so by continuity, it is positive for small \( \gamma \), which means that increasing the liquidity risk exposure above zero makes the bank better-off. On the other hand, for sufficiently high liquidity exposure, the bank profit may decrease with further increasing the exposure.

Finally, the bank net expected return sensitivity to the magnitude of the liquidity shock is,

\[
\frac{\partial \pi_{BK}}{\partial r_S} = -n^* \frac{\partial W_{BK}}{\partial r_S} = \]

\[
= n^* \Phi(-d_1^*) P_0 e^{\mu_A + \gamma r_S - q_0} \gamma > 0 \] (106)
A.3. Welfare Analysis

In the absence of hedge funds, the optimal level of deposit insurance satisfies the first order condition,

$$\sum_{n=1}^{N} \phi_n'(z^*) = c'(z^*)$$

so that

$$z^* = c'^{-1}\left(\sum_{n=1}^{N} \phi_n(z^*)\right)$$

Introducing hedge fund managers, define the function

$$F(x) = \sum_{n=1}^{N} \phi_n'(x) + \sum_{m=1}^{M} \psi_m'(x) - c'(x)$$

The optimal level of deposit insurance now satisfies the first order condition,

$$F(z^*) = 0$$

Moreover, the function $F$ is decreasing:

$$F'(x) = \sum_{n=1}^{N} \phi_n''(x) + \sum_{m=1}^{M} \psi_m''(x) - c''(x) < 0$$

where the inequality holds because each term is negative by virtue of our assumptions of concave utility functions and convex cost. Finally, we have,

$$F(z^*) = \sum_{n=1}^{N} \phi_n'(z^*) + \sum_{m=1}^{M} \psi_m'(z^*) - c'(z^*) =$$

$$= \sum_{n=1}^{N} \phi_n'(z^*) + \sum_{m=1}^{M} \psi_m'(z^*) - \left(\sum_{n=1}^{N} \phi_n'(z^*)\right) =$$

$$= \sum_{m=1}^{M} \psi_m'(z^*) > 0 = F(\bar{z}^*)$$

where we used the first order condition that defines $z^*$. Since the function $F$ is decreasing, the last equation implies that

$$\bar{z}^* > z^*$$

which proves Proposition 5.
### Table I

Summary Statistics of hedge fund style indexes. There are 151 monthly observations in the sample over the period January 1994 - August 2006.

<table>
<thead>
<tr>
<th>Index</th>
<th>Mean</th>
<th>Std.Dev</th>
<th>Min</th>
<th>Max</th>
<th>Skewness</th>
<th>Kurtosis</th>
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<tbody>
<tr>
<td>CS/Tremont HF Index</td>
<td>0.0088</td>
<td>0.0224</td>
<td>-0.0755</td>
<td>0.0853</td>
<td>0.1144</td>
<td>5.2295</td>
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<tr>
<td>Convertible Arbitrage</td>
<td>0.0072</td>
<td>0.0135</td>
<td>-0.0468</td>
<td>0.0357</td>
<td>-1.3221</td>
<td>6.0422</td>
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<tr>
<td>Dedicated Short Bias</td>
<td>0.0000</td>
<td>0.0494</td>
<td>-0.0869</td>
<td>0.2271</td>
<td>0.8052</td>
<td>4.9617</td>
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<td>Emerging Markets</td>
<td>0.0081</td>
<td>0.0468</td>
<td>-0.2303</td>
<td>0.1642</td>
<td>-0.6660</td>
<td>7.5072</td>
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<td>Equity Market Neutral</td>
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<td>0.0084</td>
<td>-0.0115</td>
<td>0.0326</td>
<td>0.3033</td>
<td>3.3027</td>
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<td>Event Driven</td>
<td>0.0093</td>
<td>0.0162</td>
<td>-0.1177</td>
<td>0.0368</td>
<td>-3.3814</td>
<td>26.6730</td>
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<td>Event Driven Distress</td>
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<td>0.0183</td>
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<td>Event Driven Multi-Strategy</td>
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<td>0.0381</td>
<td>-1.2467</td>
<td>9.4881</td>
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<td>Fixed Income Arbitrage</td>
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<td>0.0107</td>
<td>-0.0696</td>
<td>0.0205</td>
<td>-3.0593</td>
<td>19.1094</td>
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<td>Global Macro</td>
<td>0.0113</td>
<td>0.0315</td>
<td>-0.1155</td>
<td>0.1060</td>
<td>0.0204</td>
<td>5.8394</td>
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<tr>
<td>Long/Short Equity</td>
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<td>0.0294</td>
<td>-0.1143</td>
<td>0.1301</td>
<td>0.2269</td>
<td>6.7141</td>
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<td>Managed Futures</td>
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<td>0.0346</td>
<td>-0.0935</td>
<td>0.0995</td>
<td>0.0534</td>
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<td>0.0361</td>
<td>-1.1839</td>
<td>6.1828</td>
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Table II
Factor Exposures of Hedge Fund Style Indexes

Regression coefficients and t-statistics of hedge fund style index returns regressed on five common factors: the Wilshire 5000 (WLSH5K), the Lehman Brothers intermediate term credit bond index (BOND), the Goldman Sachs Commodity Index (GSCI) size factor, the U.S. Dollar Index (USDX) and the change in the spread between 3-month non-financial commercial paper and the Treasury rates (DCPTB). There are 151 monthly observations in the sample over the period January 1994- July 2006. The table reports robust standard errors. The regressions in Panel B exclude the period August-November 1998.

<table>
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<tr>
<th>Style Index</th>
<th>Const</th>
<th>WLSH5K</th>
<th>BOND</th>
<th>GSCI</th>
<th>USDX</th>
<th>DCPTB</th>
<th>R²</th>
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<tr>
<td>CS/Tremont HF Index</td>
<td>0.00356</td>
<td>0.28536</td>
<td>0.42052</td>
<td>0.06815</td>
<td>0.22842</td>
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<td>0.02200</td>
<td>0.09503</td>
<td>-0.02469</td>
<td>0.13</td>
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<td>0.23590</td>
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<td>Event Driven Distress</td>
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<td>Event Driven Multi-Strategy</td>
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<td>0.10711</td>
<td>-0.02417</td>
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Panel A: Full Sample
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