Intermediated Investment Management

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Abstract

Intermediaries such as financial advisers and funds of funds serve a vital role in the financial services industry as an interface between portfolio managers and investors. A large fraction of their compensation is often provided through rebates or kickbacks from the portfolio manager rather than directly by their clients. In a model with rational agents we provide an explanation for the widespread use of intermediaries and rebates in compensation practices. We also explore the effects of these arrangements on fund size, flows, performance and investor welfare. Intermediated funds will underperform direct channel funds based on net returns as well as gross returns. Rebates allow higher management fees to be charged, with the consequence that equilibrium fees and net returns are negatively related.

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1 Introduction

The money management industry has been recognized as having substantial influence on financial markets. The size of this industry is astounding. As evidence, the Investment Adviser Association (IAA, 2008) has estimated the total amount of assets managed by SEC-registered investment advisers at $42.3 trillion at its peak in April 2008. One consequence of the rapid growth of institutional investment is that more than 65% of US stocks are now held by institutions as opposed to individual households, as shown by US Federal Reserve Flow of Funds data.

An important reason for the tremendous size of the money management business is that there are often multiple layers of advisory services between investors and the ultimate portfolio manager. In many cases, investors do not delegate their wealth directly to money managers. They rely on intermediaries. Examples of such intermediation include financial advisers who make recommendations to their clients, brokers who sell mutual funds, and funds of funds that specialize in mutual funds or hedge funds. The prevalence of intermediation in investment management is substantial and seems to continue to increase. Bergstresser, Chalmers and Tufano (2007) document for the US that 79% of all mutual fund share classes were sold through brokers in the year 2004. According to a survey by the Investment Company Institute (ICI, 2007) only 12% of US mutual fund holders stated that they do not rely on a financial adviser. Further, Chen, Hong and Kubik (2006) document that the management of 26% of mutual funds in their sample are outsourced by fund management companies to unaffiliated advisory firms. In the hedge fund industry, about one quarter of all investments are managed by funds-of-funds according to the Hedge Fund Research database.

Fund managers often compensate intermediaries by engaging in various rebate arrangements. This may result in favorable treatment in fund distribution. A typical example of such arrangements in the case of mutual funds is that brokers receive direct compensation by sharing front-end loads, back-end loads and 12b-1 fees with the management company. Other forms of compensation are less explicit. For example, fund management companies may direct their trades to the trading desks of a brokerage firm and use the resulting trading commissions as a reward for promoting their fund to retail investors. Other forms of attempts to influence selling agents also exist. As an example, John Hancock Funds makes the following disclosures in its Statement of Additional Information: "Non-cash compensation may also take the form of occasional gifts, meals, tickets, or other entertainment as limited by NASD requirements… These payments
may provide an incentive to a Selling Firm to actively promote the Funds... You should ask your Selling firm for more information about ...marketing support payments (John Hancock Funds, 2006).

Despite their crucial role, intermediaries in the investment management industry have largely been ignored by the existing literature. Most previous studies on delegated portfolio management consider only the bilateral relationship between investors and portfolio managers. By contrast, this paper models the intermediary as a distinct agent and focuses precisely on the economic role that intermediaries play in the investment management industry. In a model with rational agents we analyze several questions related to investment management intermediation. Why is intermediation so prevalent in the investment management industry? Why is it common practice for the intermediary to be compensated by the portfolio manager instead of directly by the client? How does intermediation influence fund performance both gross and net of fees and are these results consistent with observed empirical facts? How do subsidies or kick-backs from the portfolio manager to the intermediaries affect the welfare of investors?

Several recent studies have tried to measure empirically the economic impact of intermediation in investment management. Bergstresser et al. (2007) look at the performance of mutual funds offered through the brokerage channel as compared to those offered directly to investors. They find that, even before marketing fees are deducted, risk-adjusted returns are lower for funds offered through the brokerage channel as compared to those offered directly. Chen et al. (2006) document that mutual funds managed externally significantly underperform those run internally. Ang, Rhodes-Kropf and Zhao (2008) and Brown, Goetzmann and Liang (2004) have found that fund of funds underperform average hedge funds. In all of these cases the usage of intermediation does not appear to bring economic benefits to investors.

There are also a few empirical papers that examine the potential conflicts of interest in the mutual fund distribution channels more explicitly. Edelen, Evans and Kadlec (2008) find that actively managed funds improve fund distributions by compensating their brokers with abnormally high commissions and this leads to lower fund returns. Christoffersen, Evans and Musto (2007) find that higher revenue sharing with unaffiliated broker leads to more fund inflows, and higher revenue sharing with captive brokers mitigates outflows. Chen, Yao and Yu (2007) show that mutual funds managed by insurance companies underperform

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1 E.g., Bhattacharya and Pfleiderer (1985) and Stoughton (1993).
2 Some examples of mutual fund companies that utilize direct channels include Fidelity, Vanguard and Janus. Examples of companies that offer their products through brokers include Investment Company of America, WaMu and Putnam.
their non-insurance counterparts by more than 1% per year. The authors find that this has to do with the fact that insurance funds are often cross-sold through the extensive broker/agent network of their parent firms.

Our model consists of investors, a representative investment adviser, a passively held pool of assets (e.g., index fund) and a pool of assets (active fund) run by a portfolio manager. The active fund can involve trading traditional assets (stocks and bonds) using sophisticated strategies, but also alternative assets, such as private equity, foreign currency, real assets etc., that are not present in the passive fund. Investors have heterogeneous wealth levels, and can go directly to the portfolio manager or through the indirect channel by using an adviser. In order to invest directly, investors must pay a fixed cost to identify an active manager who does not underperform the passive fund. As a result, only high networth individuals invest directly. Moreover, they earn a surplus over their reservation opportunity, which is to invest in the passive fund. Portfolio managers have market power. They are able to charge fees increasing in their expected portfolio ‘alpha’. But as the active fund has diminishing returns to size, there is an optimal amount of assets invested actively. Investment advisers also charge a fee, which compensates them for their costs of providing asset allocation services to their clients.

We first derive an equilibrium assuming that the investment advisers decisions are unsubsidized and must charge the investors their full cost in order to break even. We then extend this model by allowing the portfolio manager to subsidize the adviser through kickbacks. We solve for the optimal amount of subsidy preferred by the portfolio manager and the impact on management fees, fund sizes and flows. We then extend the analysis to the case of competition between active managers. Finally we also derive the form of an equilibrium without an adviser and compare outcomes to the cases with subsidized or unsubsidized advisers, respectively.

Our major findings are as follows. First, we rationalize the widespread use of investment advisers. They facilitate the participation of small investors in actively managed portfolios by economizing on information costs. Second, portfolio managers always have incentives to influence the asset allocation decisions of advisers through subsidies. Therefore we explain the widespread use of side payments as a method for compensating advisers. Surprisingly, when asset allocation is influenced by kickbacks from the portfolio manager to advisers, the usage of advisory services increases. Third, intermediated funds underperform
direct channel funds because they are larger in scale. Side payments accentuate this result because of their effect in raising fees. Fourth, funds of funds underperform the average fund itself as a result of the feature that net returns of funds do not include the search cost of identifying superior performance. Fifth, our model predicts negative correlation between performance of actively managed funds and fees charged by fund managers. This is because aggressive subsidy payments, i.e. kickbacks to the intermediaries allow the fund manager to maintain the fund size even in the presence of high management fees. Finally we find that competition amongst active managers reduces the extent of subsidies, thereby improving measured fund performance.

A seminal paper on the subject of investment management is the AFA Presidential address of Sharpe (1981). He analyzes the coordination failure in the presence of multiple portfolio managers. Recently, this model has been extended by van Bindsbergen, Brandt and Koijen (2008). By contrast, we consider the role of an intermediary between the client and the portfolio manager.

Several other papers have examined the structure of the investment management industry. Mamaysky and Spiegel (2002) and Gervais, Lynch and Musto (2005) provide a theory for the existence of fund families. Massa (1997) presents a model to explain the market segmentation and fund proliferation in the mutual fund industry. Grundy (2005) develops a model in which it is optimal to employ an intermediary, the investment bank, in order to achieve the optimal fund size. The bank can do this when its advantage in terms of resolving information asymmetry outweighs the additional contracting imperfections. By contrast, in our model, a competitive adviser resolves the information asymmetry problem completely given a known return on the passive asset. Ding (2008) looks at the relationship between money managers and the brokerage firms they trade through. The focus of that paper is on the `soft dollars' relationship in terms of trading activities rather than on the intermediation between clients and money managers, as in our paper.

An interesting paper exploring the role of kickbacks in the medical field is that of Pauly (1979). He considers a medical practitioner who is able to engage in `fee-splitting' practices with a specialist. He finds that there is no point in prohibiting such practices in a fully competitive environment because services are provided at marginal cost. However, when there are market imperfections such as monopoly or incomplete pricing of insurance, fee-splitting can actually improve client welfare.

In the next section we set up the basic model, with behavioral assumptions on the three participants
in the game: the investors, the investment adviser and the portfolio manager. In section 3 we derive the form of the equilibrium without the possibility of subsidization by portfolio managers. Section 4 derives the impact of kickbacks from the portfolio manager to the advisers. Our model is generalized to imperfect competition between managers is section 5. Section 6 considers the form of the equilibrium in a situation where advisory services are not available. Section 7 compares the outcome and welfare in the three alternative scenarios. Section 8 concludes the paper.

2 Model Setup

In this section we describe the agents, their behavior and how they interact. There are three classes of agents in the model: (1) the active manager; (2) the set of investment advisers, modeled as a representative agent; (3) the pool of investors in the economy.

2.1 Assets and Managers

There are two types of assets in which investors can invest. First, there is a passive fund, such as an index fund, with an expected gross return \( r_f \) (i.e., one plus the rate of return). If investors are risk averse and all use the same model of risk premia in pricing assets, then we could describe \( r_f \) as a `risk-adjusted' expected return. However to simplify description of the problem, we refer to this as an expected return, as if investors were risk-neutral. Both investors and advisers can invest in the passive fund without cost.

The second type of asset is an active fund, whose expected return (once again risk-adjusted) is equal to \( r_p \). The active portfolio manager utilizes her expertise in managing these assets. However, because of market impact or limited applicability of the portfolio manager’s expertise, we assume decreasing returns to scale in the amount of investment. Specifically, we assume that

\[
    r_p = \alpha - \gamma A, \tag{1}
\]

where \( \alpha \) represents the initial expected return (assumed to be greater than the passive return) and \( \gamma \) is a coefficient representing the rate at which returns decline with respect to the aggregate amount of funds, \( A \), that are placed with the portfolio manager. For a discussion of this assumption see Berk and Green (2004)
In addition to investing in the passive asset and obtaining returns equal to $R_f$, the investors can choose to delegate their portfolio decisions to an investment adviser who advises multiple clients, or they can decide to invest directly with the portfolio manager. Because there are potentially many active managers who would produce inferior returns relative to the passive fund _ex ante_, we assume that there is a fixed screening cost, $C_0$, that must be expended by an investor in order to identify potentially valuable active managers. Therefore a direct investor pays the cost $C_0$ and then is able to identify a manager whose fund returns on the first dollar under management are superior, i.e. for whom $\alpha > R_f$. Investors decide optimally whether or not to pay this fixed cost. Alternatively they can avoid it by delegating their funds to the adviser in which case he makes the investment choice for them.

The investment adviser has the expertise required to ascertain potentially superior portfolio managers. However the adviser incurs a cost that increases in the amount of assets allocated to the active manager. For simplicity we assume a constant marginal cost, $c_A$, that is incurred at the end of the period and is proportional to the amount of capital allocated to the active fund. An example of such a cost faced by investment advisers is the cost of retaining legal counsel for assistance in the event of lawsuits. Another example of such costs are regulatory requirements, increasing with the number of clients, such as ensuring compliance with fiduciary responsibilities concerning client qualification. We can regard these costs as the insurance premium required as a function of the amount invested in actively managed assets. The adviser can also allocate money to the passive fund costlessly.

Finally we describe the nature of the fees that are charged by the active manager and the investment adviser. We assume that the adviser charges a proportional fee, $f_A$, based on the end-of-period value of actively managed assets. This fee is determined endogenously in the model due to competition among advisers. In fact we assume perfectly competitive behavior so that the fee satisfies a zero-profit condition. This also implies that the adviser is not able to charge for the funds allocated to the passive fund, since the investors obtain no benefit to using him in this case. The portfolio manager also charges a proportional

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3 Empirical evidence supporting this effect can be found in Chen, Hong, Huang and Kubik (2004) and Ang et al. (2008).
4 Alternatively, investors may have different endowment risks and may have to expend the cost in order to ascertain the correlation between the active manager’s portfolio and their endowment.
5 In a model in which individual investors incur additional costs of investing in the passive fund on their own, the adviser could charge a fee on total assets under management.
fee, \( f_p \), on the end-of-period value of the assets managed.

The investors therefore have to decide amongst three investing strategies. If they invest in the passive fund themselves, they get a return of \( R_f \). If they invest directly with the portfolio manager, they pay their fixed information collection fee as well as the portfolio management fee. If they delegate their decision to the adviser, they have to pay two management fees for the actively invested portion of their holdings: both that of the adviser and that of the portfolio manager. The active manager can receive funds directly from the investors (the direct channel) or indirectly through the investment adviser (the indirect channel).

### 2.2 Investors Behavior

Assume that each investor has wealth \( x + C_0 \), where \( x \) follows a Pareto distribution with the following probability density function:

\[
f(x) = \frac{kA_m^k}{x^{k+1}}, \quad k > 1,
\]

where \( A_m > 0 \) denotes the minimum wealth level (net of the search cost \( C_0 \)). The Pareto distribution has been widely used to describe the distribution of wealth among individuals. Empirical studies have found that this distribution characterizes actual wealth distributions fairly well, except for its properties at the lower end. An important feature of this distribution is that the probability density \( f(x) \) decreases monotonically in wealth, implying that the fraction of wealthy investors is relatively small while the fraction of investors with low levels of wealth is relatively large. The parameter \( k \) characterizes the extent of wealth equality. Complete equality of wealth is characterized by \( k \rightarrow \infty \), while \( k \rightarrow 1 \) corresponds to complete inequality. Pareto's original estimate of \( k \) based on income data clustered around 1.5. Later estimates of this parameter ranged from 1.6 to 2.4 for income distributions (Champernowne (1952)) and from 1.3 to 2.0 for wealth distributions (Yntema (1933)).

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6Our general conclusion does not depend on this specific assumption about the wealth distribution. In previous versions, we have utilized several different assumptions about the wealth distribution; our results are qualitatively similar. Including \( C_0 \) in the initial wealth simplifies the subsequent notation.

7See Persky (1992) for a brief review of this literature.

8The Gini coefficient, a widely used measure of inequality, for the Pareto distribution is given by \( \frac{1}{2k-1} \).
We standardize the population to be 1. Therefore the total wealth available for investment is

\[ W = \int_{A_m}^{\infty} xf(x)dx + C_0 = \frac{kA_m}{k-1} + C_0, \quad k > 1. \tag{3} \]

Based on their wealth level, investors can choose whether to invest directly or indirectly. Let \( A_D \) and \( A_I \) denote the amounts of direct and indirect investment to the active fund. Therefore the total amount of money under active management is \( A = A_D + A_I \) and the rate of return of the actively managed portfolio is \( R_p = \alpha - \gamma(A_D + A_I) \).

Investors take returns as given and have no market power, i.e., they are atomistic and do not take into account the diseconomy of scale in active portfolio management when they decide where to channel their funds. Therefore, the amount of capital invested in the actively managed fund via the adviser will adjust until investors earn their reservation rate, \( R_f \). Thus, investing through the adviser is identical to investing in the passive fund. An investor with wealth \( A^* + C_0 \) will be indifferent between contracting directly with the portfolio manager and getting a net rate of return \( R_p (1 - f_p) \) and investing via the adviser, where \( A^* \) satisfies the following condition:

\[ [\alpha - \gamma(A_D + A_I)](1 - f_p)A^* = R_f(A^* + C_0), \]

i.e.,

\[ A^* = \frac{C_0 R_f}{(1 - f_p)[\alpha - \gamma(A_D + A_I)]} - R_f. \tag{4} \]

It is obvious that all investors whose wealth is smaller than \( A^* + C_0 \) will prefer to invest via the adviser whereas those with wealth greater than \( A^* + C_0 \) will prefer to contract directly with the portfolio manager. We therefore refer to this latter set of investors as 'high networth individuals'. Indeed there is evidence to this effect. In \( \text{ICI (2004)} \) it is documented that customers who purchase funds through a brokered mutual funds sales force channel as opposed to the direct channel are less wealthy (with median income of $93,800 v. $101,300). They also have lower median financial assets ($363,700 v. $447,900).

Given that all investors with wealth levels greater than \( A^* + C_0 \) invest directly, we can solve for the
amount of money channeled directly to the portfolio manager:

\[ A_D = \int_{A^*}^{\infty} xf(x)dx = \frac{kA_m^k}{(k - 1)(A^*)^{k-1}}. \]  

(5)

Note that if \( A^* = A_m \), then \( A_D = W - C_0 \), and all investors would contract with the portfolio manager directly. To make our analysis interesting, we assume that if all wealth net of the search cost \( C_0 \) is invested in the active portfolio, the return of the active portfolio will be lower than the reservation rate of the investors, i.e.,

\[ \alpha - \gamma(W - C_0) < R_f. \]  

(6)

3 Investment Management Equilibrium

We now solve for the equilibrium in our model of investment management. First we discuss the behavior of the adviser and subsequently the behavior of the portfolio manager. In this section, we assume that there are no 'kickbacks' to the adviser so that his decision-making is uninfluenced in that it is based entirely on the respective asset returns. In the next section, we allow for kickbacks from the portfolio manager to the advisers.

3.1 Advisers Behavior

Since the representative fund adviser charges a proportional advisory fee \( f_A \), based on the end-of-period value of the active fund investment in his clients' accounts, he solves the following problem:

\[ \max_w w(f_A[\alpha - \gamma(A_D + A_I)](1 - f_P) - c_A). \]  

(7)

where \( w \) represents the portfolio weight of the active asset. When solving this problem, the adviser takes the returns on the active fund as given as well as the proportional advisory fee.

Therefore, a finite solution requires that the maximand in (7) equals zero or

\[ f_A[\alpha - \gamma(A_D + A_I)](1 - f_P) - c_A = 0. \]  

(8)
In addition, the fact that investors are indifferent to using the adviser implies

\[ [\alpha - \gamma(A_D + A_I)](1 - f_p)(1 - f_A) - R_f = 0. \]  \hspace{1cm} (9)

Substituting (9) into (8) gives the following equilibrium condition:

\[ [\alpha - \gamma(A_D + A_I)](1 - f_p) = R_f + c_A. \]  \hspace{1cm} (10)

Notice that (10) implies that the net-return of the actively managed portfolio exceeds the return on the passive asset by exactly the marginal cost of advisory services. If the net-return of the actively managed portfolio would be below this threshold value, nobody would invest actively using the adviser. If it would be above, the advisor would invest everything in the actively managed portfolio which would depreciate its expected return.\footnote{Note w in the solution to (7) is indeterminate because it does not matter in our model whether the funds invested passively are held on account with the adviser or by the investors themselves. However, the total amount of wealth invested actively through the adviser is determinate and solved for below.}

Equations (8) and (9) can also be used to determine the equilibrium fee charged by the adviser. Substituting and solving for \( f_A \) yields

\[ f_A = \frac{c_A}{R_p(1 - f_p)} = \frac{c_A}{R_f + c_A}. \]  \hspace{1cm} (11)

In other words, the fee compensates the advisor for the cost incurred.\footnote{Note that \( c_A \) must be “discounted” by \( R_p(1 - f_p) \), since it is proportional to the assets allocated at the beginning of the period whereas the advisory fee is proportional to the end-of-period value of the portfolio.}

In order to determine which investors will invest via the direct channel, we substitute equation (10) into equation (4), we have

\[ A^* = \frac{c_0R_f}{c_A}. \]  \hspace{1cm} (12)

Notice that this threshold level of wealth does not depend on \( \alpha \). Therefore the investors can optimally decide whether or not to collect information in the equilibrium without knowing the portfolio managers' potential ability, \( \alpha \).
Substituting equation (12) into equation (5) then gives the equilibrium amount of money invested directly:

\[ A_D = \frac{kA_m^kC_A^{k-1}}{(k - 1)(C_0R_f)^{k-1}}. \]  

(13)

The most important property of this relationship is that the amount of money invested directly is independent of the fees of the portfolio manager. This is because of the competitive nature of the adviser. If the active manager attempts to increase her fees, for the same gross rate of return, investors would reduce the amount of capital that is allocated to active management via the adviser. Therefore the same net rate of return is achieved by active investing and thus there is no effect on the marginal direct investor or the aggregate amount of money invested directly. This property is critical to understanding the model and will be exploited below.

3.2 Portfolio Manager Behavior

Now we describe the problem faced by the portfolio manager. From equation (10) we know that total funds under active management satisfy

\[ A_D + A_I = (\alpha - \frac{R_f + c_A}{1 - f_p}) \frac{1}{\gamma}. \]  

(14)

That is, funds under management are related to the difference between \( \alpha \) and the grossed-up return on the passive asset plus the marginal advisory cost, scaled by the depreciation factor, \( \gamma \). The manager now takes this into account and solves the following fee maximization problem:

\[
\max_{f_p} [\alpha - \gamma(A_D + A_I)](A_D + A_I)f_p
\]

subject to equation (14).

Substituting the constraint into the objective function, the manager's problem is

\[
\max_{f_p} \left[ \frac{R_f + c_A}{1 - f_p} \right] \left[ \alpha - \frac{R_f + c_A}{1 - f_p} \right] \frac{1}{\gamma} f_p.
\]
It is easy to solve this for the optimal portfolio manager fees, which we record as a proposition.

**Proposition 1.** The optimal portfolio manager fee in the investment management equilibrium without fee rebates is

\[
f_p' = \frac{\alpha - R_f - c_A}{\alpha + R_f + c_A}.
\]

(16)

Now using this result, we can determine how the fee is impacted by \( \alpha \). Taking the partial derivative of equation (16),

\[
\frac{\partial f_p}{\partial \alpha} = \frac{2(R_f + c_A)(\alpha + R_f + c_A)^2}{(\alpha + R_f + c_A)^2} > 0.
\]

Hence the fees are increasing in the managerial ability. Further we can solve for the active manager's profit:

\[
\Pi_p = \frac{(\alpha - R_f - c_A)^2}{4\gamma},
\]

(17)

and the total assets allocated to the manager:

\[
A_I + A_D = \frac{\alpha - R_f - c_A}{2\gamma}.
\]

(18)

Substituting equation (13) into equation (18), we have

\[
A_I = \frac{\alpha - R_f - c_A}{2\gamma} - \frac{kA_m^{k-1}}{(k - 1)(C_0R_f)^{k-1}}.
\]

(19)

We assume that \( C_0 \) is big enough, or \( \gamma \) is small enough, to ensure that \( A_I \) in (19) is positive, i.e., not all investment into the active portfolio comes from the direct channel. 11

In summary the investment management equilibrium features positive profits of the portfolio manager and zero profit of the investment adviser. Returns on the actively managed portfolio net of management fees are greater than those of the passive fund. Rates of return earned by direct investors in the active fund

11 If this condition would not hold, the adviser is not utilized at all. In this case, the analysis in section 6 would apply.
exceed those earned by indirect investors. Nevertheless only the high net worth individuals find it optimal to invest directly.

4 Fee Rebates

We now extend the model to consider the aspect of rebates or 'kickbacks' from the portfolio manager to the investment adviser. The idea here is that the portfolio manager desires to influence the decisions of the investment adviser, so that the fund is accessed by small investors to a greater extent. The purpose of this part of our model is to evaluate the impact of such activities on asset returns and fund flows.

Rebates can take various forms, either explicit or implicit. The most common type of a rebate involves the load charge of a mutual fund. In this case the fund charges a sales fee and then rebates a large part back to the sales agent (broker). In practice actual loads are based on a declining scale related to the magnitude of the investment in a fund. There are front-end loads, back-end loads and 12b-1 fees, which are incurred on a continuing basis. Bergstresser et al. (2007) document the nature of these charges for different investment channels, and find that the charges are much higher for brokered sales as compared to direct sales.

4.1 Impact on Fund Flows

We choose to model the manager's attempt to influence the advisers in the following way. The kickback is specified ex ante and paid at the end of the period by the portfolio manager to the adviser in the form of rebate, $\delta$, proportional to the value of the portfolio component allocated to the active fund.

As a result of the rebate, an advisor will invest in the active fund up to the point where its net return satisfies

$$f_A[\alpha - \gamma(A_D + A_I)](1 - f_p) - c_A + \delta = 0.$$  \hspace{1cm} (20)

The same condition (9) must still hold even with fee rebates. Therefore we arrive at the following equilibrium rate of return condition:

$$[\alpha - \gamma(A_D + A_I)](1 - f_p) = R_f + c_A - \delta.$$  \hspace{1cm} (21)
Moreover the competitive advisers reduce their fees, compared to the case without rebates. The new advisory fee is equal to

\[ f_A = \frac{c_A - \delta}{R_f + c_A - \delta}. \]  

(22)

It is straightforward to show that the fee decreases as \( \delta \) increases.\(^{12}\)

As in the case without subsidy, we can substitute equation (21) into (4) and we find that the threshold wealth level becomes

\[ A^* = \frac{C_0 R_f}{c_A - \delta}. \]  

(23)

and the amount invested directly becomes

\[ A_D = \frac{k A_m (c_A - \delta)^{k-1}}{(k - 1)(C_0 R_f)^{k-1}}. \]  

(24)

We see first by comparing respectively equations (23) with (12) and (24) with (13) that the impact of the subsidy changes the marginal investor who is indifferent between investing directly and indirectly to one with a higher wealth level. Consequently the amount of funds invested directly decreases due to the subsidy. Hence a key result is that kickbacks shift investors from the direct investment channel to the indirect investment channel. This occurs because the portfolio manager increases her fees to finance the rebate to the advisor. As investors are shifted into the indirect channel, we will see that they suffer a welfare loss because they are now pushed down to their reservation utility. As before, though, the portfolio manager’s fee is constrained by the asset allocation decision of the adviser and the amount of direct investment.

### 4.2 Optimal Rebates

We now endogenize the rebate by allowing the portfolio manager to choose her optimal \( \delta \). The total amount of compensation equals the amount of indirect investment in the active portfolio times the proportional

\(^{12}\)This relation is consistent with the reality that some advisors differentiate themselves as ‘fee-only’ advisers and advertise their independence, which allows them to charge higher advisory fees. At the other extreme, there exist brokers who receive their entire compensation from load rebates and charge no other advisory fees.
rebate, \( A_1 \delta \). The portfolio manager will maximize her profit net of the kickback payments. Therefore, her problem is

\[
\max_{f_p, \delta} \quad \Pi_p = [\alpha - \gamma(A_D + A_I)](A_D + A_I)f_p - A_I\delta
\]

subject to the constraints (21) and (24). We can now solve for the optimal portfolio manager fees and kickback payments imparted to the investment adviser.

**Proposition 2.** When rebates are feasible, the optimal fee charged by the portfolio manager is

\[
f_p^* = \frac{\alpha - R_f - c_A + 2c_A/k}{\alpha + R_f + c_A}.
\]  
(25)

The optimal ratio of subsidy from the portfolio manager to the investment adviser is

\[
\delta^* = \frac{c_A}{k}.
\]  
(26)

**Proof.** Substituting the constraints into the objective function, we see that the first order condition for the optimal fee is

\[
f_p^* = \frac{\alpha + 2\delta - R_f - c_A}{\alpha + R_f + c_A}.
\]  
(27)

Substituting this expression back into the objective function and using again constraint (21), we get

\[
\Pi_p = \frac{(\alpha - R_f - c_A)^2}{4\gamma} + A_D\delta.
\]  
(28)

Comparing this expression with equation (17), we see that the portfolio manager’s profit in the new equilibrium is simply her profit in the equilibrium without kickbacks plus the loss of remaining direct investors due to lower fund return. Setting the derivative of \( \Pi_p \) with respect to \( \delta \),

\[
\frac{\partial \Pi_p}{\partial \delta} = \frac{kA_m^k(c_A - \delta)^{k-2}}{(k-1)(c_0R_f)^{k-1}}(c_A - k\delta),
\]
equal to zero, we get the optimal subsidy stated in the proposition.\footnote{If \( k > 2 \), \( \delta = c_A \) also satisfies the first order condition. However, in this case, \( A_D = 0 \); the portfolio manager's profit is not maximized. Therefore \( \delta = c_A \) is not optimal.}

The second order conditions can be verified in a straightforward fashion.

Proposition 2 shows that there is an interior optimal amount of subsidy. The intuition for this result can be obtained by recognizing that an increasing subsidy is associated with higher management fees, as embodied in equation (25). This has distinct effects for the following three categories of investors. First, the manager does not enjoy the benefits from a higher fee from individuals who are already investing indirectly since they are fully compensated via kickbacks. Second, the benefits of a higher fee are fully captured by the portfolio manager in the case of a direct investor. Third, the portfolio manager loses a discrete amount on those investors who switch from investing directly to investing indirectly in response to the fee increase. This latter effect dominates as the number of direct investors decreases.

Effectively subsidizing the adviser permits the portfolio manager to price discriminate between large and small investors, while charging the same management fee. Once high networth investors have paid the fixed cost, they have a lower elasticity of demand, compared to the indirect investors, whose adviser faces a positive marginal cost. Therefore the portfolio manager would optimally like to charge higher fees to the high networth investors, without adversely affecting the demand of small investors.

From equation (26), we can easily see that the optimal subsidy is increasing in the advisory cost \( c_A \) and decreases in \( k \), which measures the degree of equality of the wealth distribution. The advisory cost \( c_A \) represents the maximum subsidy that can be provided before all investors leave the direct channel. Therefore, not surprisingly, the optimal subsidy is increasing in \( c_A \).

The relation between the optimal subsidy and \( k \) is also intuitively appealing. When \( k \) is large, there are fewer high networth investors, therefore, the portfolio manager does not extract much surplus by providing a subsidy. By contrast, when \( k \) is close to 1, the fraction of high networth investors is relatively large. As a result, the portfolio manager has a stronger incentive to subsidize to be able to extract their surplus.

To further analyze the impact of optimal fee rebates, we compute the equilibrium size of the fund. Substituting the optimal fee given by equation (27) into (21), we find that the fund size in the equilibrium...
with the kickback is

\[ A_D + A_I = \frac{\alpha - R_f - c_A}{2\gamma}. \]

Note that this is exactly the same as the fund size in the equilibrium without the kickback, equation (14). Intuitively this occurs because in both cases, the indirect investors are marginal investors in the sense that a slight decrease in net returns would lead them to switch to the passive portfolio. Thus their net return, \( R_f \), is constant and independent of the kickback. Since the advisory services market is competitive, the portfolio manager has to cover the cost of advisory services and therefore the marginal cost of obtaining one dollar from the indirect investor is \( R_f + c_A \), which is also independent of kickback. When the portfolio manager optimizes the fund size, she equates the marginal gross portfolio return from attracting an additional dollar of indirect investment with this marginal cost. Both the marginal return and marginal costs of the portfolio manager at the optimal fund size is independent of the kickback payments and therefore the fund size is identical in both cases.

Rebates, therefore, allow the portfolio manager to extract some surplus of the large investors. In this process, some investors will be switching from the direct to the indirect investment channel. Indeed, substituting the optimal \( \delta^* \) in equation (26) into (24), we see that the active investment through the direct channel in the equilibrium with rebates is

\[ A_D = \frac{kA_m^k(c_A - c_A/k)^{k-1}}{(k - 1)(C_0 R_f)^{k-1}}, \]  

which is strictly smaller than \( A_D \) without rebates as embodied in equation (13).

We record the observations from this section in the form of a proposition.

**Proposition 3.** In the investment management equilibrium where advisers are subsidized, the active fund size is the same as when advisers are unsubsidized. However, more investors utilize advisory services by investing indirectly and the net return of the active fund is lower. The portfolio manager charges a higher fee, while advisers charge a lower fee but receive a compensatory kickback from the portfolio manager.

Since we have shown that fund size is constant, while fees are increased because of rebates, our model predicts lower performance for actively managed mutual funds that use greater levels of rebates. This
conforms to some recent evidence from mutual funds where rebates are derived from excess commissions paid to brokerage firms [Edelen et al., 2008]. Also our results predict that the portfolio management fee has a one-for-one negative impact on the fund’s net return, which is consistent with the finding of Carhart (1997). Our model shows these empirical patterns can result from conflicts of interest in the distribution channel for funds.

A striking result of our analysis is that despite the potential conflicts of interest associated with the rebates, investment advisers are actually used to a greater extent in equilibrium than when there are no rebates. The reason is that the portfolio manager optimally raises her fees, which makes direct investment less attractive. The adviser is forced to lower his own fees in order to remain competitive with the alternative asset. Hence, the adviser’s asset management business becomes larger.

5 Competition in Active Management

Now we introduce an environment where there is more than one active manager. For simplicity in this section we consider two managers competing for funds under management. Our results are robust to a finite number of managers.14

A central issue in the case of competition is whether it increases or decreases the subsidy imparted to the asset allocation choices of advisers. One hypothesis is that competition disciplines managers and induces lower kickbacks; the opposite hypothesis holds that competition creates stronger incentives for the managers to provide kickbacks to advisers. In this case they are in a race to outdo each other.

5.1 Unsubsidized Adviser

We assume that the two managers are pursuing similar active strategies. The managers are symmetric with respect to ability and therefore both have potential abnormal returns, \( \alpha \). The diseconomy of scale occurs with respect to aggregate funds under active management, implying that the gross return, \( R_{Pi} \), of portfolio manager \( i \) is given by

\[
R_{Pi} = \alpha - \gamma \sum_{i=1}^{2} A_i;
\]

\(^{14}\)The results for \( N \) managers are available from the authors.
where $A_i \equiv A_{Di} + A_{ii}$ is the size of the fund managed by manager $i$ (both direct and indirect investment).

We employ the concept of Cournot-Nash competitive strategies with respect to fund sizes. In this case, each manager optimizes her fund size taking as given the size the other fund:

$$\max_{A_i} \Pi_i = (\alpha - \gamma \sum_{i=1}^{2} A_i) A_i f_P,$$  \hspace{1cm} (30)

subject to essentially the same condition as in equation (8) considered earlier on the behavior of the investment advisers:

$$(\alpha - \gamma \sum_{i=1}^{2} A_i)(1 - f_P) = R_f + c_A.$$  \hspace{1cm} (31)

Substituting for $f_P$ in the objective function using the constraint, and considering the first order conditions for both managers simultaneously, we obtain the following proposition.

**Proposition 4.** The Cournot-Nash equilibrium involving competition amongst fund managers is unique and symmetric and involves the equilibrium fund sizes

$$A^*_i = \frac{\alpha - R_f - c_A}{3\gamma}, \quad i = 1, 2.$$  \hspace{1cm} (32)

The equilibrium management fee is given by

$$f^*_P = \frac{\alpha - R_f - c_A}{\alpha + 2R_f + 2c_A}.$$  \hspace{1cm} (33)

Proposition 4 therefore shows that while the fund size of an individual manager is smaller than before (equation (14)), the aggregate funds under active management is equal to

$$A_1 + A_2 = \frac{2(\alpha - R_f - c_A)}{3\gamma},$$  \hspace{1cm} (34)

which is greater than in the monopolistic case. Further fees are lower (equation (33) as compared to (16)). Even though the fund size is larger and the fees are lower, net returns on active portfolios are the same as before.
5.2 Subsidized Advisers

Now we consider the Cournot-Nash equilibrium in which each portfolio manager provides a subsidy, \( \delta \), to the adviser. In this case the portfolio managers' optimization problem can be written as

\[
\max_{A_D, A_I} \Pi = \left[ \alpha - \gamma \sum_{i=1}^{2} (A_{Di} + A_{Ii}) \right] (A_{Di} + A_{Ii}) f_P - A_{Ii} \delta,
\]

subject to the constraints

\[
\left[ \alpha - \gamma \sum_{i=1}^{2} (A_{Di} + A_{Ii}) \right] (1 - f_P) = R_f + c_A - \delta,
\]

\[
A_D = \sum_{i=1}^{2} A_{Di} = \frac{k A_m^{k} (c_A - \delta)^{k-1}}{(k - 1)(C_0 R_f)^{k-1}}.
\]

The solution to this problem is provided in the following proposition.

Proposition 5. The solution to the Cournot-Nash competition game between two portfolio managers who can influence the investment advisers through kickbacks is unique and symmetric. The equilibrium features the same total fund size as in the case without subsidies. However the allocation through the indirect channel is larger and the direct channel is smaller. The optimal amount of subsidy is given by

\[
\delta^* = \frac{c_A}{2k - 1}.
\]

and the optimal fee schedule for each manager is given by

\[
f^*_P = \frac{\alpha - R_f + 3c_A/(2k - 1)}{\alpha + 2R_f + 2c_A}.
\]

Proof. See Appendix A.1

This proposition shows that our results on the impact of fee rebates are carried through in the case of (imperfect) competition between multiple portfolio managers. As before the amount of funds actively managed is not affected by such activities. Not surprisingly we find that more funds are managed actively when there is more competition. However it is not true that there is a `race in outdoing' the other manager.
in terms of subsidies. In fact they optimally select a lower degree of subsidy to the advisers as compared to the monopolistic case. The reason has to do with the direct investors. Increasing the subsidy through kickbacks financed by increasing fees implies that one fund will lose high networth investors compared to a competitor that does not follow suit. Therefore, even if the fund could gain access to more indirect investors, it winds up losing direct investors and this effect dominates. As a result, competitive forces actually counteract the tendency to subsidize the advisers.

6 Equilibrium Without Advisers

In order to investigate the role of investment advisers in delegated portfolio management, we now examine an equilibrium in which investment advisers do not exist. When there is no investment adviser, the only vehicle for active investing is directly through the portfolio manager. As a result, \( A_i = 0 \); the fund size is determined solely by \( A_D \). The portfolio manager maximizes her profit by choosing an optimal fee and fund size. Therefore, the manager’s problem can be written as:

\[
\max_{A_D, f_P} \Pi_P = (\alpha - \gamma A_D) A_D f_P
\]  

subject to

\[
A_D = \frac{k A_m^k}{(k - 1)(A^*)^{k-1}}, \\
(\alpha - \gamma A_D)(1 - f_P) A^* = R_f(A^* + C_0),
\]

where the second constraint ensures that the marginal investor is indifferent between investing with the portfolio manager and the passive fund. We can see from equation (41) that there is an inverse relation between fund size and fees charged by the portfolio manager. Therefore either variable can be optimized by the manager equivalently.

Note that the informational assumptions made here are somewhat stronger than those needed previously with the investment adviser. Recall that the decision whether to invest directly did not depend in equilibrium on the potential value of active management, \( \alpha \), when the investment adviser is present. Now we must assume that the direct investor knows in advance which \( \alpha \) will obtain, after the cost, \( C_0 \), is ex-
This, of course does not violate our earlier justification for the cost, since without paying it, there would be an adverse selection problem borne by the investor.

Problem (40) is now solved in the next proposition.

Proposition 6. In the portfolio management equilibrium without advisers, there exists a unique interior optimal fund size and management fee, which are the solutions to the following set of equations:

\[ \alpha - R_f - 2\nu A_D - \frac{\lambda k}{k-1} A_D^{1/(k-1)} = 0, \quad (42) \]

and

\[ f_p = \frac{\alpha - \nu A_D - (R_f + \lambda A_D^{1/(k-1)})}{\alpha - \nu A_D}, \quad (43) \]

where

\[ \lambda \equiv C_0 R_f \left( \frac{k-1}{kA_m} \right)^{1/(k-1)}. \quad (44) \]

Proof. See Appendix A.2.

A general analytical solution to the equation system in Proposition 6 for an arbitrary \( k \) is not available. Appendix A.3 provides the solutions for two special values of \( k \) which are close to the empirically observed values: \( k = 1.5 \) and \( k = 2 \). For general values of \( k \) the solutions can be found using numerical methods.

In the following section we compare outcomes for the three scenarios: no advisers, unsubsidized advisers, and subsidized advisers. In particular we address the question whose interests investment advisers really serve: the investors' or the portfolio manager's. The key question is how investors are impacted by the presence of the adviser. We also consider the consequence of rebates on aggregate welfare of investors and the portfolio manager.
7 Comparison of Equilibria

The comparison between the equilibria with unsubsidized and subsidized advisers is relatively easy to make. However, it is less obvious how equilibrium outcomes change when there is no adviser. To illustrate the differences, we construct a numerical example and solve for the equilibria in the three cases: (1) an unsubsidized investment adviser; (2) a subsidized investment adviser; and (3) no investment adviser.

7.1 Outcomes

In this subsection we analyze the equilibrium fund size, the amount of direct investment, the management fee and net returns as a function of \( k \). The results are illustrated in figure 1.

We plot the fund size in the three equilibria in figure 1a. Here the solid line represents the fund size in both the subsidized- and unsubsidized-adviser equilibria since they are the same. As one can see from the graph, the size of the active portfolio is substantially larger in the presence of investment advisers, especially when \( k \) is large, i.e., when the fraction of high networth investors in the economy is small. This is because investment advisers assist small investors to invest in the active portfolio.

Note that before-fee returns are inversely related to fund size. Figure 1a therefore implies that the before-fee performance is higher for funds that are sold only through the direct channel as compared to funds that are sold through both channels. This is supported by empirical evidence in Bergstresser et al. (2007), who show that returns are higher for direct channel funds as compared to brokered funds even before marketing fees are deducted.

Figure 1b compares the amount of investment through the direct channel. In the case without advisers, this is equivalent to the total fund size. While the fund size is important for the portfolio manager’s profit, the amount of direct-channel investment is critical for the investor welfare since only the direct investors earn a surplus over their reservation return. In all scenarios direct investment decreases as \( k \) increases, i.e. when there are fewer high-networth individuals. Not surprisingly, the direct-channel investment is smallest in the presence of subsidized advisers because some investors are shifted to the indirect channel due to the kickback.

We now turn to the effect of the existence of unsubsidized advisers on the amount of direct investment in the active fund. There are two effects to consider. First, for given management fees, the absence of
This figure compares the outcomes of three different equilibria. Solid lines correspond to the equilibrium with unsubsidized advisers (in panel (a), it also corresponds to the equilibrium with subsidized advisers), dashed lines correspond to the equilibrium with subsidized advisers, while dotted lines correspond to the equilibrium without advisers. The values of parameters other than $k$ are as follows: $R_f = 1.05, \alpha = 1.1, \gamma = 0.5 \times 10^{-9}, A_m = 5 \times 10^7, C_0 = 5 \times 10^7, c_A = 0.02$. 
advisers increases the amount of direct investment, as there is no substitute. However, there is a second effect due to the endogeneity of the management fee. In the absence of advisers, it is optimal for the portfolio manager to charge higher fees since demand is more inelastic. The magnitude of this second effect increases when there are more high-networth investors, which corresponds to a small $k$. The second effect dominates the first in this region.

Figure 1c illustrates the portfolio manager's fees as a function of $k$. In the case with unsubsidized advisers, the fee is constant. This is because the indirect investors are the marginal ones, and their reservation return is independent of their wealth. Moreover, the fee charged in the case of unsubsidized advisers is by far the lowest of the three scenarios. The reason for the low fee is that the presence of the adviser effectively makes the demand more elastic and thus fee reductions are more profitable for the portfolio manager.

If the adviser can be subsidized, then price discrimination between high networth and low networth investors becomes possible, since part of the management fee can be kicked back to the indirect investors via the adviser. Thus the higher fee charged by the portfolio manager only impacts the direct investors. As $k$ becomes larger, i.e. as the relative number of high networth investors becomes smaller, the potential benefit of price discrimination decreases. Therefore the optimal fee is smaller.

Figure 1d compares the return of the active portfolio after management fee. Consistent with the results on the difference in fees, the net return is always lower in the case of kickbacks as compared to the situation with unsubsidized advisers. The behavior of net returns in the absence of the adviser is related to the direct investment decision of high networth investors. When there are many of them ($k$ small) the portfolio manager can capitalize by increasing her fees to a greater extent. As a result the net returns are reduced below the unsubsidized case. When there are fewer of them ($k$ large) the portfolio manager is only able to sell to lower networth investors by reducing her fees, therefore the net returns are more attractive than in the unsubsidized case.

### 7.2 Welfare Analysis

We now analyze how the aggregate welfare is affected by the presence of advisers, as well as by fee rebates. Investment advisers (when in existence), as well as the indirect investors always have zero surplus. We can
therefore define the total welfare as the sum of the portfolio manager's profit and the surplus of the direct investors, where investor surplus is defined relative to the default of not paying the search cost, investing in the passive asset and earning net return $R_f$.

The effect of rebates on investor welfare has already been described in section 4. Recall that fee rebates shift some investors from the direct channel to the indirect channel. Investors who are forced to switch to the indirect channel lose their surplus, while those who remain in the direct channel get a lower net return as the portfolio manager raises her fee. The total effect on investor surplus is therefore unambiguously negative. Not surprisingly, the portfolio manager's profit changes in an opposite direction. As we have seen from equation (28), the portfolio manager's profit increases by $A_D C_A / k$ after introducing the kickback.

Combining the profit for the portfolio manager with the surplus earned by the investors we compute the total welfare in the three scenarios. The details of these computations are carried out in appendix A.4. We are able to prove the following proposition:

**Proposition 7.** The levels of total welfare in equilibria with no advisers, $U^0$, unsubsidized advisers, $U^1$, and subsidized advisers, $U^2$, are given respectively by

$$U^0 = (\alpha - \nu A^0_D - R_f) A^0_D - \theta^0 C_0 R_f,$$

$$U^1 = A^1_D C_A - \theta^1 C_0 R_f + \Pi^1_p,$$

$$U^2 = A^2_D (c_A - \delta) - \theta^2 C_0 R_f + \Pi^2_p,$$

where $A^i_D = \frac{k A^i_m}{(k-1)(A^i_A)^{k-1}}$ denotes the amount of assets invested into the active portfolio directly, $\theta^i \equiv (\frac{A^m_i}{A^i_A})^k$ denotes the fraction of investors choosing the direct channel, $i = 0$ (no adviser), $i = 1$ (unsubsidized adviser) and $i = 2$ (subsidized adviser). Furthermore, we have

$$U^1 - U^2 = (A^1_D - A^2_D) C_A - (\theta^1 - \theta^2) C_0 R_f > 0,$$

i.e., the total welfare is strictly higher in the equilibrium with unsubsidized advisers than in the equilibrium with subsidized advisers.

*Proof.* See Appendix A.4

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This figure compares welfare in three different equilibria. Solid lines correspond to the equilibrium with unsubsidized advisers, dashed lines correspond to the equilibrium with subsidized advisers, while dotted lines correspond to the equilibrium without advisers. The values of parameters other than $k$ are as follows: $R_f = 1.05$, $\alpha = 1.1$, $\gamma = 0.5 \times 10^{-9}$, $A_m = 5 \times 10^7$, $C_0 = 5 \times 10^7$, $c_A = 0.02$. 

Figure 2: Welfare comparison across equilibria
Figure 2a plots the total welfare in the three equilibria under the same parameter values used to plot figure 1. Consistent with Proposition 7, the total welfare in the equilibrium without kickbacks (the solid line) is always higher than in the equilibrium with kickbacks (the dashed line). When kickbacks are permissible the equilibrium will feature excessive use of investment advisers. This occurs as some clients are being induced to use the proportional cost technology of the advisers when the fixed-cost technology, i.e. the direct channel would be more efficient.

Importantly, the figure also shows that both of these equilibria dominate the no-adviser equilibrium: the total welfare in the equilibrium without advisers (the dotted line) is lower than in the other two equilibria for all different values of $k$ we consider. This provides a rationale for the existence of advisory services in facilitating capital investment through active funds.

Figure 2b compares the profits of the portfolio manager in three equilibria for various values of $k$. With the presence of unsubsidized investment advisers, the profit of the portfolio manager is independent of the wealth distribution parameter $k$, therefore it is a horizontal line in the diagram. The portfolio manager is strictly better-off in the equilibrium with kickbacks. She benefits more from paying kickbacks when $k$ is smaller. This is because she extracts more rents from the high networth investors when there are many such investors in the economy.

For most reasonable values of $k$, the portfolio manager also benefits from the presence of investment advisers, even when kickbacks are forbidden. This is not surprising because the existence of investment advisers allows the portfolio manager to provides her services to small investors, who will otherwise not participate in the active portfolio. Interestingly, this is not always the case. When $k$ is small, the portfolio manager's profit is higher in the no-adviser equilibrium than in the equilibrium with unsubsidized advisers, indicating that when there are many wealthy investors, the portfolio manager may be better off by declining investment through an indirect channel.

Figure 2c plots the investor surplus in different equilibria, as derived in Appendix A.4. Consistent with our analytical results, investor surplus is higher when advisers are unsubsidized than when they are subsidized. The figure also shows that from the investors' point of view, subsidized advisers are always worse than no advisers at all. More interestingly, when $k$ is large, even unsubsidized advisers can reduce investor welfare. Only when $k$ is small, investors are better off with unsubsidized advisers than without advisers.
This is consistent with the shape of the net return of the active portfolio plotted in figure 1d.

In summary, our analysis shows that investment advisers, even if they can be influenced by the portfolio manager, improve total welfare of the portfolio manager and investors. However, although investment advisers claim to serve investors, their presence mainly benefits the portfolio manager. Even when investment advisers are unsubsidized, they improve investor welfare only if the fraction of high networth investors in the economy is large. If they can be influenced by the portfolio manager, then investors as a whole are better off without them.

8 Conclusions

The market for financial products and services is expanding rapidly as corporations and financial institutions package cash flows and contingent claims in different ways. As the number of alternative investment opportunities placed before investors has increased, investment advisers play a critical role in allocating assets. Investment advisory services are employed by many types and categories of investors, including retail investors, corporate pension funds, university endowment committees and many other institutional investors. The purpose of this paper has been to investigate the effect of such financial intermediaries on investors' portfolio decisions, fund returns and fund management fees.

A unique feature of our model is that it provides a rationale for the use of advisory services by small investors. They are able to participate in an actively managed portfolio, consisting for instance of alternative investments which would not be economical without the use of an adviser. Our analysis shows that although small investors use the adviser, large investors are the ones who may benefit or be hurt. The prospect of attracting an additional pool of small investors via the adviser may induce the portfolio manager to lower his fees aggressively, which benefits the large investors. However, access to the pool of small investors also increases the size of the actively managed portfolio. Due to diseconomies of scale, this lowers the fund return before management fee. If this effect dominates, then the large investors are worse off with financial intermediation. The portfolio manager unambiguously benefits from the presence of financial intermediaries. Furthermore, financial intermediation allows the portfolio manager to capture rents from large investors if the adviser can be subsidized by the portfolio manager. In this case even more investors utilize advisers. Thus, in our model the usage of advisory services increases when there is a potential for
conflicts of interest.

We derive a number of predictions that are consistent with empirical evidence. First, brokered funds underperform direct channel funds; second, portfolios managed by intermediaries, such as funds-of-funds, underperform other actively managed portfolios; third, funds distributed by intermediaries that are more heavily subsidized by the portfolio managers, such as insurance mutual funds, high-load funds, and funds paying abnormally high soft-dollars to improve fund distribution, underperform other funds.

Our model also generates some empirical predictions that have not been tested yet. For example, we show that the effect of intermediation by independent advisers on the net fund return depends on the distribution of investors' wealth in the economy. Our model also predicts that the incentive of the portfolio manager to subsidize the adviser increases when the fraction of large investors in the economy increases. Furthermore, we show that competition between active managers lowers the equilibrium subsidy therefore resulting in higher net fund returns. Finally, our model implies that the importance of indirect sales through investment advisers increases with fund size. Ceteris paribus, large funds sell a larger fraction through advisers whereas small funds feature proportionally more direct investors.

Another justification for the use of advisers pertains to potential behavioral biases of small investors. In this case, their use may avoid problems of loss aversion, overconfidence and myopia, etc. As long as the advisory business is sufficiently competitive, our major results will be preserved. However, behaviorally biased small investors are also likely to have limited information about the capability of advisers and this may lead to market power of the advisers that could offset the gains. We leave these issues for future research.

Our results point out that without kickbacks advisers improve total welfare of investors and portfolio managers. When advisers can be influenced through kickbacks, then investors' welfare is unambiguously reduced. However, apparent policy implications for regulating the investment advisory services industry must be interpreted very cautiously. Even though investors are worse off with subsidized advisers, the portfolio manager and investors, taken together, are better off compared to not having investment advisers. Furthermore, in a more general model the value created by active managers would be endogenous. If their potential profit is curtailed by regulation, they are less likely to make the investment necessary to attain high levels of expertise.
A Appendix

A.1 Proof of Proposition 5

Proof. Substituting out \( f_p \) and \( \delta \) in the objective function using the two constraints, we have

\[
\Pi_{P_i} = \left[ \alpha - R_f - c_A - \gamma \sum_{i=1}^{2} (A_{Di} + A_{II}) \right] (A_{Di} + A_{II}) + A_{Di}(c_A - \frac{C_0 R_f}{[KA_{m0}^{k} (k-1)A_0]^{1/(k-1)}}),
\]

The first order conditions are

\[
\frac{\partial \Pi_{P_i}}{\partial A_{II}} = \left[ \alpha - R_f - c_A - \gamma \sum_{i=1}^{2} (A_{Di} + A_{II}) \right] - \gamma (A_{Di} + A_{II}) = 0, \quad i = 1, 2, \quad (49)
\]

\[
\frac{\partial \Pi_{P_i}}{\partial A_{Di}} = \left[ \alpha - R_f - c_A - \gamma \sum_{i=1}^{2} (A_{Di} + A_{II}) \right] - \gamma (A_{Di} + A_{II})
+ \left( c_A - \frac{C_0 R_f}{[KA_{m0}^{k} (k-1)A_0]^{1/(k-1)}} \right) - \frac{A_{Di}}{A_D (k-1)} \frac{C_0 R_f}{[KA_{m0}^{k} (k-1)A_0]^{1/(k-1)}}, \quad i = 1, 2. \quad (50)
\]

Substituting equation (49) into (50), the latter reduces to

\[
c_A = \left[ 1 + \frac{A_{Di}}{A_D (k-1)} \right] \frac{C_0 R_f}{[KA_{m0}^{k} (k-1)A_0]^{1/(k-1)}}, \quad i = 1, 2. \quad (51)
\]

Note that the total size of each individual fund, \( A_i = A_{Di} + A_{II} \), is fully determined by equation (49). It is unique, symmetric, and independent of the kickback payments. Therefore equations (32) and (34) remain to hold. Equation (51) shows that the direct channel choice is also unique and symmetric across managers. Therefore \( A_{Di} = A_D / 2 \) in equation (51). Notice further that from (37) we have

\[
\frac{C_0 R_f}{[KA_{m0}^{k} (k-1)A_0]^{1/(k-1)}} = c_A - \delta,
\]

thus we end up with

\[
c_A = \left( 1 + \frac{1}{2(k-1)} \right) (c_A - \delta)
\]

at optimum. Therefore the optimal subsidy is given in equation (38). Substituting the optimal fund size and
optimal subsidy into equation (36), we obtain equation (39). □

A.2 Proof of Proposition 6

Proof. Combining the two constraints in problem (40) we immediately obtain equation (43). Substituting this expression back into the objective function and differentiating, we have

\[
\frac{\partial \Pi_P}{\partial A_D} = (\alpha - R_f) - 2yA_D - \frac{\lambda k}{k - 1} A_D^{1/(k-1)},
\]

\[
\frac{\partial^2 \Pi_P}{\partial A_D^2} = -2y - \frac{\lambda k}{(k - 1)^2} A_D^{(2-k)/(k-1)} < 0.
\]

Equation (42) is given by setting the first order condition above equal to zero. Since \( \Pi_P \) is strictly concave when \( A_D > 0 \), the first order condition is both necessary and sufficient condition for the solution to this maximization problem; furthermore, the optimal \( A_D \) is unique. To prove the existence of an interior solution, \( 0 < A_D < W - C_0 \), to the first order condition, note that \( \frac{\partial \Pi_P}{\partial A_D} > 0 \) if \( A_D = 0 \). Due to the monotonicity of the first derivative, it suffices to show this derivative becomes negative as \( A_D \to W - C_0 \), i.e., as \( A_D \) converges to the aggregate wealth of the economy net of the search cost \( C_0 \). This is guaranteed by the boundary condition (5).

A.3 Two Special Cases

An analytical solution to the first order condition for the no-adviser equilibrium, equation (42), is not available for an arbitrary \( k \). We provide the solutions for two special values of \( k \) which are close to the empirically observed values: \( k = 1.5 \) and \( k = 2 \). We can show that the optimal fund size chosen by the portfolio manager (via the optimal fee) for \( k = 1.5 \) and \( k = 2 \) respectively, is

\[
A_D = \begin{cases} 
\frac{\alpha - R_f}{2y + C_0 R_f / \kappa_0} & \text{if } k = 2, \\
(\sqrt{\gamma^2 + C_0 R_f (\alpha - R_f) / (3 \kappa_0^3)} - \gamma) \frac{3 \kappa_0}{C_0 R_f} & \text{if } k = 1.5.
\end{cases}
\]

Accordingly, the optimal fee is

\[
f_P = \begin{cases} 
1 - \frac{R_f + C_0 R_f A_D/(2 \kappa_0^2)}{\alpha - \mu A_D} & \text{if } k = 2, \\
1 - \frac{R_f + C_0 R_f A_D^2/(9 \kappa_0^3)}{\alpha - \mu A_D} & \text{if } k = 1.5.
\end{cases}
\]
and the return of the active portfolio net of the management fee is

\[
R_p(1 - f_p) = \begin{cases} 
R_f + \frac{C_0 R_f A_0}{2A_m^2} & \text{if } k = 2, \\
R_f + \frac{C_0 R_f A_0^2}{3A_m^3} & \text{if } k = 1.5.
\end{cases}
\]

(54)

The optimal fund size can be derived easily from the first order condition, while the optimal fee and the net return of the active portfolio follow from the two constraints in problem (40).

A.4 Proof of Proposition 7

Proof. In the case without investment advisers, the number of direct investors is the same as the number of investors investing in the active portfolio. Denote the total surplus of direct investors, relative to the default of passive investment, by \(S^0_D\), we have

\[
S^0_D = \int_{A^0_0}^{+\infty} \left[ x(\alpha - \nu A^0_D)(1 - f_p) - (x + C_0)R_f \right] f(x) \, dx
\]

\[
= \left[ (\alpha - \nu A^0_D)(1 - f_p) - R_f A^0_D - \theta^0 C_0 R_f \right] A^0_D
\]

where \(A^0_0\) is the threshold wealth level (net of the search cost \(C_0\)) that makes the marginal investor indifferent between the passive fund and investing with the active portfolio manager, \(A^0_D = \int_{A^0_0}^{+\infty} xf(x) \, dx = \frac{k A^k_m}{(k-1)(A^0_c)^{k-1}}, \quad \theta^0 \equiv \int_{A^0_0}^{+\infty} f(x) \, dx = \left( \frac{A^m_m}{A^0_c} \right)^k\).

Correspondingly, the total welfare can be written as:

\[
U^0 = S^0_D + \Pi^0
\]

\[
= S^0_D + (\alpha - \nu A^0_D) A^0_D f_p
\]

\[
= (\alpha - \nu A^0_D - R_f) A^0_D - \theta^0 C_0 R_f.
\]

In the equilibrium with unsubsidized advisers, the after-fee return of the active fund is equated to \(R_f + c_A\) by the advisers' actions. Therefore the direct investor's surplus, \(S^1_D\), is given by

\[
S^1_D = \int_{\frac{c_A}{R_f}}^{+\infty} \left[ x(R_f + c_A) - (x + C_0)R_f \right] f(x) \, dx
\]

\[
= A^1_D c_A - \theta^1 C_0 R_f,
\]

where \(A^1_D\) is derived in equation (13), and \(\theta^1 \equiv \left( \frac{c_A A^m_m}{C_0 R_f} \right)^k\).
Similarly, since the after-fee return of the active portfolio in the case with kickback equals \( R_f - \delta \), the total surplus of the direct investors in the kickback equilibrium is given by

\[
S_D^2 = A_D^2(c_A - \delta) - \vartheta^2 C_0 R_f,
\]

where \( A_D^2 \) is derived in equation (29), and \( \vartheta^2 \equiv \left( \frac{(c_A - \delta) A_m}{C_0 R_f} \right)^k \).

Adding the manager's profit to the investor surplus, we get the total welfare \( U_1 \) and \( U_2 \) in Proposition 7.

To compare the total welfare in equilibria with and without kickback, first note that allowing kickbacks reduces the investors' surplus by

\[
S_D^1 - S_D^2 = A_D^2 \delta + (A_D^1 - A_D^2) c_A - (\vartheta^1 - \vartheta^2) C_0 R_f.
\]

Furthermore, recall that from equation (28), we know that allowing kickbacks increases the portfolio manager's profit by \( A_D^2 \delta \). Combining these two welfare effects, we have

\[
U^1 - U^2 = (A_D^1 - A_D^2) c_A - (\vartheta^1 - \vartheta^2) C_0 R_f
= \int_{C_0 R_f/c_A}^{C_0 R_f/c_A} x f(x) \left[ c_A - \frac{C_0 R_f}{x} \right] dx > 0.
\]

The above expression is positive for any \( \delta > 0 \), since \( c_A - C_0 R_f/x \) is positive for any \( x \) not less than \( C_0 R_f/c_A \).

Therefore kickbacks always decrease total welfare. Our proof also makes it clear that in the equilibrium with kickbacks, the welfare loss of the investors staying in the direct channel is exactly offset by the gain of the portfolio manager. The loss of investors who would originally choose the direct channel, but are forced to switch to the indirect channel because of the kickback, is the deadweight loss of total welfare.

\( \square \)
References

Ang, A., M. Rhodes-Kropf and R. Zhao (2008), "Do Funds-of-Funds Deserve Their Fees-on-Fees?" *Journal of Investment Management, 6*, 1–25.


