Permanent vs. Transitory Idiosyncratic Shocks in Flood & Rose:
An Example
11/3/04

The point of this note is to clarify through an example the relationship between the
covariance assumptions F&R make, and the covariance assumptions made normally in asset-
pricing Finance models.   We make no distribution assumptions and make no specific
assumptions about the marginal rate of substitution, m_{t+1}. It will follow that we can't
actually compute any of the relevant covariances, but we really do not need closed forms for
this point.

From the pricing equation

$$p_t^j = E_t(m_{t+1}, x_{t+1}^j)$$

We have

$$\frac{x_{t+1}^j}{z_t^j} = \delta_t [\frac{p_t^j}{z_t^j} - COV(m_{t+1}, \frac{x_{t+1}^j}{z_t^j})] + \varepsilon_{t+1}^j$$

where z is a deflator that will take on several definitions below. Other symbols are defined in
the text.

So far, nothing new. Now, however, we make some assumptions about the time series
processes for prices.

 (This note will produce **sufficient but not necessary** conditions to produce our
orthogonality condition. Just to anticipate, if we can keep the RHS regressor out of the
covariance term completely we should be comfortable with our assumption – it will give
consistent estimates.)

Also, to make life easy, let's suppose that the assets in question do not pay dividends – this is
easy to fix, but notation intensive.

Following Fama and French, JPE, April '88 "Permanent and Transitory Components of
Stock Prices" we model the log of stock prices as follows:

$$p_t = p_t^A + p_t^I$$

where p_t^A is the aggregate part of log price and p_t^I is the idiosyncratic part. (F&F do not have the aggregate/idiosyncratic split.) We have suppressed the j (firm) superscript as it plays no role here. The prices are understood to be individual stock prices.

For this example we need mess with only the idiosyncratic part, which is split as:

$$p_t^I = W_t + T_t,$$

where *W* is a random walk or permanent component and *T* is a transitory component – the *I* superscript is suppressed for now. (This is the F&F split)

These two components follow:

$$W_t = W_{t-1} + a_t$$

and

$$T_t = \rho T_{t-1} + b_t, \quad -1 \le \rho \le 1$$

where *a* and *b* are white.

Now, suppose we set up **two** filters.

The first is the one we have played with already – it splits price growth into aggregate and idiosyncratic components.

The second one will take the idiosyncratic parts and split them into permanent and transitory components.

(There are a number of papers on this – all with different splitting rules and different data. Fama and French (cited above) is the best known. John Cochrane has a paper in the QJE February 1994, and Dupuis and Tessier, have an even more recent working paper "The US Stock Market and Fundamentals: A Historical Decomposition" Bank of Canada WP 2003-20.)

The line is that for high-frequency returns, e.g., monthly, the aggregate innovation variance split is between 35% transitory (F&F), 57% transitory (JC) and 70% transitory (the Canadian guys). The Canadian guys find that higher frequency gives a greater variance percentage to the transitory. (Note that we need idiosyncratic variance splits, not aggregate variance splits, but I doubt this is important for our purposes.) This is a slightly charged literature – Larry Summers got it going a bit with his over-reaction work. The idea is that stock market over-reactions are inherently transitory – by definition.

Once we have done all of our filtering, we build the synthetic price:

$$\tilde{p}_t = p_t^A + W_t + \hat{\rho}T_t .$$

The trick – for this example – is that we have replaced T_t **by** $\hat{\rho}T_t$. These are all logs, so our data deflator is $\exp(\tilde{p}_t) = \exp(p_t^A + W_t + \hat{\rho}T_t)$. This deflation allows us to remove this period's transitory shock from the synthetic return. (As always, the synthetic return is not so synthetic. It is equivalent to a standard return on portfolio of size $\exp((1-\hat{\rho})T_t)$

Our synthetic return is

$$\frac{\exp(p_{t+1}^j)}{\exp(\tilde{p}_t^j)} = \frac{\exp(p_{t+1}^A + W_{t+1} + \rho T_t + b_{t+1})}{\exp(p_t^A + W_t + \hat{\rho}T_t)} = \exp(p_{t+1}^A - p_t^A + a_{t+1} + b_{t+1}) .$$

This return is cool. It has been engineered so that it contains **none of this periods' idiosyncratic shock** – a_t and b_t have been removed from the synthetic return and from the return part of $COV_t(m_{t+1}, \frac{\exp(p_{t+1}^j)}{\exp(\tilde{p}_t^j)})$. (We need to be careful since we have set $(\rho - \hat{\rho})T_t = 0$. I think this is ok, but I am a little worried because T is an estimate, not a fixed number.)

It follows that for the deflator we use, the only way the idiosyncratic shock could possibly get into $COV(,)$ is through m. Our maintained condition thus becomes the orthogonality of the idiosyncratic shock and m.

Now we construct the regressor. It is

$$\frac{\exp(p_t^j)}{\exp(\tilde{p}_t^j)} = \frac{\exp(p_t^A + W_t + T_t)}{\exp(p_t^A + W_t + \hat{\rho}T_t)} = \exp((1-\hat{\rho})T_t) .$$

(This always happens in our stuff. The regressor is the same as the portfolio size of standard returns.)

Our regression is:

$$\exp(p_{t+1}^A - p_t^A + a_{t+1} + b_{t+1}) = const + \delta_t \exp((1-\hat{\rho}^j)T_t^j) + error .$$

The term *error* is made up of the rational expectations error, which is orthogonal to T_t

plus $COV_t(m_{t+1}, \dfrac{\exp(p_{t+1}^j)}{\exp(\tilde{p}_t^j)})$, which has had all traces of T_t removed. We end up, therefore, with just what we want.

The advantage of this scheme is that it uses a weaker orthogonality condition than we have used in our current paper while preserving our two tricks – the engineered deflator and the idiosyncratic regressor. It also reconciles the relation between our orthogonality condition and the one used in most asset-pricing Finance papers.