Estimating the Expected Marginal Rate of Substitution: A Systematic Exploitation of Idiosyncratic Risk

Andrew K. Rose and Robert P. Flood

All materials (data sets, output, papers, slides) at:

http://faculty.haas.berkeley.edu/arose

Two Objectives:

1. Derive new methodology to estimate and compare the expected marginal rate of substitution (EMRS)
2. Illustrate technique empirically, and assess integration of assets across markets

The Paper in a Nutshell

1. Idiosyncratic Shocks are expected to earn expected intertemporal marginal rate of substitution (EMRS)
2. There are LOTS of idiosyncratic shocks
   - Noise is good, since it can be exploited

Definition of Asset Integration

- Assets are *integrated* if satisfy asset-pricing condition:

$$p_t^j = E_t(m_{t+1} x_{t+1}^j) \quad (1)$$

- Completely standard general framework
- Note that m_{t+1} is the same for all j

Paper Focus: $E_t(m_{t+1})$

- Conditional Mean of Marginal Rate of Substitution/Stochastic Discount Factor/Pricing Kernel/risk-free rate/zero-beta return ties together all intertemporal decisions
- Subject of much research (Hansen-Jagannathan, etc.)
- Prices all assets (and intertemporal decisions!)
- Unobservable, even *ex post* (but estimable)

Key:

- Should be identical for all assets *in an integrated market*

Motivation: Who Cares about Integration and EMRS

- MRS is "DNA" of intertemporal economics
- Appears in RBC, new-Keynesian, and in between
- Whenever agents maximize an intertemporal utility function, MRS is used

Standard Macroeconomics

- Appears in IS curves that link interest rates and inflation

$$\frac{1}{(1+i_t)} = E_t\left(\frac{\rho u_c(c_{t+1}) * q_t}{u_c(c_t) * q_{t+1}}\right)$$

Bonds/IS Curve

- Links prices with future firm revenues

$$p_t = E_t\left(\frac{\rho u_c(c_{t+1}) * q_t}{u_c(c_t) * q_{t+1}} * x_{t+1}\right)$$

Stocks/Investment

In both Equations

$$m_{t+1} = \frac{\rho u_c(c_{t+1}) * q_t}{u_c(c_t) * q_{t+1}}$$

- Is bond pricing integrated with stocks/investment-pricing?
- What arguments belong in IS curve?
- If stock and bond pricing are not integrated, different MRS with possibly different arguments.

International Finance

$$\frac{s_t}{(1+i_t)} = E_t(m_{t+1}^* * s_{t+1})$$

Foreign-currency Bond, or

$$1 = E_t(m_{t+1}^* * \frac{s_{t+1}(1+i_t^*)}{s_t})$$

Can rewrite as:

$$1 = COV_t(m_{t+1}^* * \frac{s_{t+1}(1+i_t^*)}{s_t}) + E_t m_{t+1}^* E_t(\frac{s_{t+1}(1+i_t^*)}{s_t}$$

If domestic- and foreign-currency pricing is integrated,

$$E_t(m^*_{t+1}) = E_t m_{t+1} = \frac{1}{(1+i_t)} \quad \text{then}$$

$$1 = COV_t(m^*_{t+1} * \frac{s_{t+1}(1+i^*_t)}{s_t}) + \frac{(1+i^*_t)}{(1+i_t)} E_t(\frac{s_{t+1}}{s_t})$$

With lack of integration, however,

$$E_t(m^*_{t+1}) \neq E_t m_{t+1} \quad \text{then}$$

$$1 = COV_t(m^*_{t+1} * \frac{s_{t+1}(1+i^*_t)}{s_t}) + \frac{(1+i^*_t)}{(1+i_t)} \frac{E_t m^*_{t+1}}{E_t m_{t+1}} E_t(\frac{s_{t+1}}{s_t})$$

$$\theta_t = \frac{E_t m^*_{t+1}}{E_t m_{t+1}} \qquad \text{is stochastic without integration}$$

- Interpretation: domestic-currency bonds have higher liquidity return than foreign-currency denominated bonds.

- Rejection of UIP due ONLY to risk premium correlations?
  - Or is $\theta_t \neq 1$ a factor also?

Summary: Why Should we Care about EMRS?

- o Links interest rates to inflation
- o Links prices with future firm revenues
- o Links leisure today with leisure tomorrow
- o Links domestic and foreign asset prices (UIP deviations)…

- MRS of serious intrinsic interest

Empirical Strategy

- Stocks have lots of noise and big cross-sections

Definition of Covariance/Expectation Decomposition:

$$p_t^j = E_t(m_{t+1}x_{t+1}^j) = COV_t(m_{t+1}, x_{t+1}^j) + E_t(m_{t+1})E_t(x_{t+1}^j). \tag{2}$$

Rearrange and substitute actual for expected x (WLOG):

$$x_{t+1}^j = -[1/E_t(m_{t+1})]COV_t(m_{t+1}, x_{t+1}^j) + [1/E_t(m_{t+1})]p_t^j + \varepsilon_{t+1}^j,$$

$$x_{t+1}^j = \delta_t(p_t^j - COV_t(m_{t+1}, x_{t+1}^j)) + \varepsilon_{t+1}^j \tag{3}$$

where $\delta_t = 1/E_t(m_{t+1})$ and $\varepsilon_t \equiv x_{t+1}^j - E_t(x_{t+1}^j)$

3 Assumptions Traditionally Made for Estimation:

1) *Rational Expectations*: ε_{t+1}^{j} is assumed to be white noise, uncorrelated with information available at time t,

2) *Factor Model*:

$COV_t(m_{t+1}, x_{t+1}^{j}) = \beta_j^0 + \Sigma^i \beta_j^i f_t^i$, for the relevant sample,

3) *Risk-Free Rate*: Use Treasury-bill return for $E_t(m_{t+1})$

Three Approaches

An Asset Pricing/Factor Model is:

$$x_{t+1}^{j} = \delta_t (p_t^j + \sum_i \beta^{i,j} f_t^{\,i}) + \varepsilon_{t+1}^{j} \tag{4}$$

Traditional Finance Asset Pricers: Use all 3 assumptions

- Normalize (4) by dividing by p_t^j

$$x_{t+1}^{j} / p_t^{j} - (1 + i_t) = (\Sigma_i \beta^{i,j} f_t^{i}) + \varepsilon_{t+1}^{j}$$

- Delivers "good" estimates of factor loadings (β)
- Oriented towards estimating risk premia
- But no/poor estimates of $E_t(m_{t+1})$
    - It's simply equated to T-bill! (alternatives implausible/imprecise)

Flood-Rose (2003): Make first 2 assumptions

- Normalize (4) by dividing by p_{t-1}^{j}

$$x_{t+1}^{j} / p_{t-1}^{j} = \delta_t (p_t^{j} / p_{t-1}^{j} + \Sigma_i \beta^{i,j} f_t^{i}) + \varepsilon_{t+1}^{j}$$

- Can estimate EMRS (non-linearly)

- Still need factor model

    - Thus rejection of equal EMRS across markets is conditional on asset pricing model; reject joint hypothesis (integration PLUS asset pricing/factor model)

Our New Approach

- Normalize (4) by dividing by "systematic price" $\tilde{p}_t^j$, defined as p_t^j with idiosyncratic part set to zero.
- Delivers estimates of EMRS, but no factor loadings at all!

- Normalizing by $\widetilde{p}_t^j$ delivers:

$$x_{t+1}^j / \widetilde{p}_t^j = \delta_t[(p_t^j / \widetilde{p}_t^j) - COV_t(m_{t+1}, x_{t+1}^j / \widetilde{p}_t^j)] + \varepsilon_{t+1}^j$$

- First part (inside brackets) is an idiosyncratic function.
- Second part (covariance) a function of aggregate phenomena.
    - Can therefore be ignored (as part of residual) without affecting consistency of $\delta_t = 1 / E_t(m_{t+1})$

Can estimate parameters of interest without covariance model!

- Adding Covariance (factor) model would improve efficiency of estimating $\{\delta_t\}$
  - Potential Cost is inconsistency (mis-specified covariance model)

Notes

- Focus is on exploiting (not ignoring) idiosyncratic risk
  - Idiosyncratic risk carries no risk premium
  - Our test involves estimating and comparing costs of carrying purely idiosyncratic risk
- We don't model covariances with factor model
  - Instead substitute model of aggregate returns plus orthogonality condition

Strengths of the Methodology

1. Based on general intertemporal model
2. Do not model/parameterize MRS (with e.g., utility function/consumption data); it varies arbitrarily
3. Requires only accessible, reliable data on prices, returns
4. Can be used at all frequencies

5. Can be used for all types of assets

6. No special software required

7. Focus is on intrinsically interesting object, namely expectation of marginal rate of substitution (EMRS)

Empirical Implementation

- Cannot observe $\tilde{p}_t^j$; must use observable empirical counterpart to systematic price, denoted $\hat{p}_t^j$.

- We use OLS to estimate J (= # assets) time-series regressions:

$$\ln(p_t^j / p_{t-1}^j) = a_j + b_j * \ln(\bar{p}_t / \bar{p}_{t-1}) + c_j^1 f_t^1 + c_j^2 f_t^2 + c_j^3 f_t^3 + v_t^j$$

where $\bar{p}_t$ is market-wide average price

- Can then compute:

$$\hat{p}_t^j \equiv p_{t-1}^j * \exp(\hat{a}_j + \hat{b}_j \ln(\overline{p}_t / \overline{p}_{t-1}) + \hat{c}_j^1 f_t^1 + \hat{c}_j^2 f_t^2 + \hat{c}_j^3 f_t^3)$$

- No special attachment to this model; just need *some* model

Estimation

- Equation to be estimated is linear:

$$x_{t+1}^{j} / \hat{p}_{t}^{j} = \delta_{t} (p_{t}^{j} / \hat{p}_{t}^{j}) + u_{t+1}^{j}$$

- May have non-trivial measurement error (hence inconsistency), also generated regressor (hence incorrect standard errors)

- GMM (using lags, $\{p_t^j / \hat{p}_{t-1}^j\}$, as set of IVs) solves both problems; OLS too

Data Sets

- Decade of monthly data (1994M1-2003M12)
- Year of daily data (2003)
    - Could use different frequencies too
- American data from CRSP; Canadian (in $) from DataStream
    - End-of-period prices and returns (with dividends)
    - Use only firms with full span of data (selection bias?)
- Could use bonds/other assets …

Monthly Data Set: 120 observations

- 380 firms from S&P 500 traded on NYSE
- 150 Firms from S&P/TSE index

Daily Data Set: 247 Business Days (both markets open)

- 440 firms from S&P 500 traded on NYSE
- 220 Firms from S&P/TSE index

Results

- Start with 380 firms from S&P 500 in Figure 1 (117 monthly observations; lose observations because of lead/lag)
- First estimate EMRS with only 190 firms
  - Plot mean, +/-2 standard error confidence interval
- 2 different estimation methods (OLS, GMM)
  - similar results

What Does EMRS, $\{\hat{\delta}\}$, Look Like?

- Reasonable Mean (slightly over unity)
- Tight confidence intervals (estimation precision)
- Lots of time series volatility!

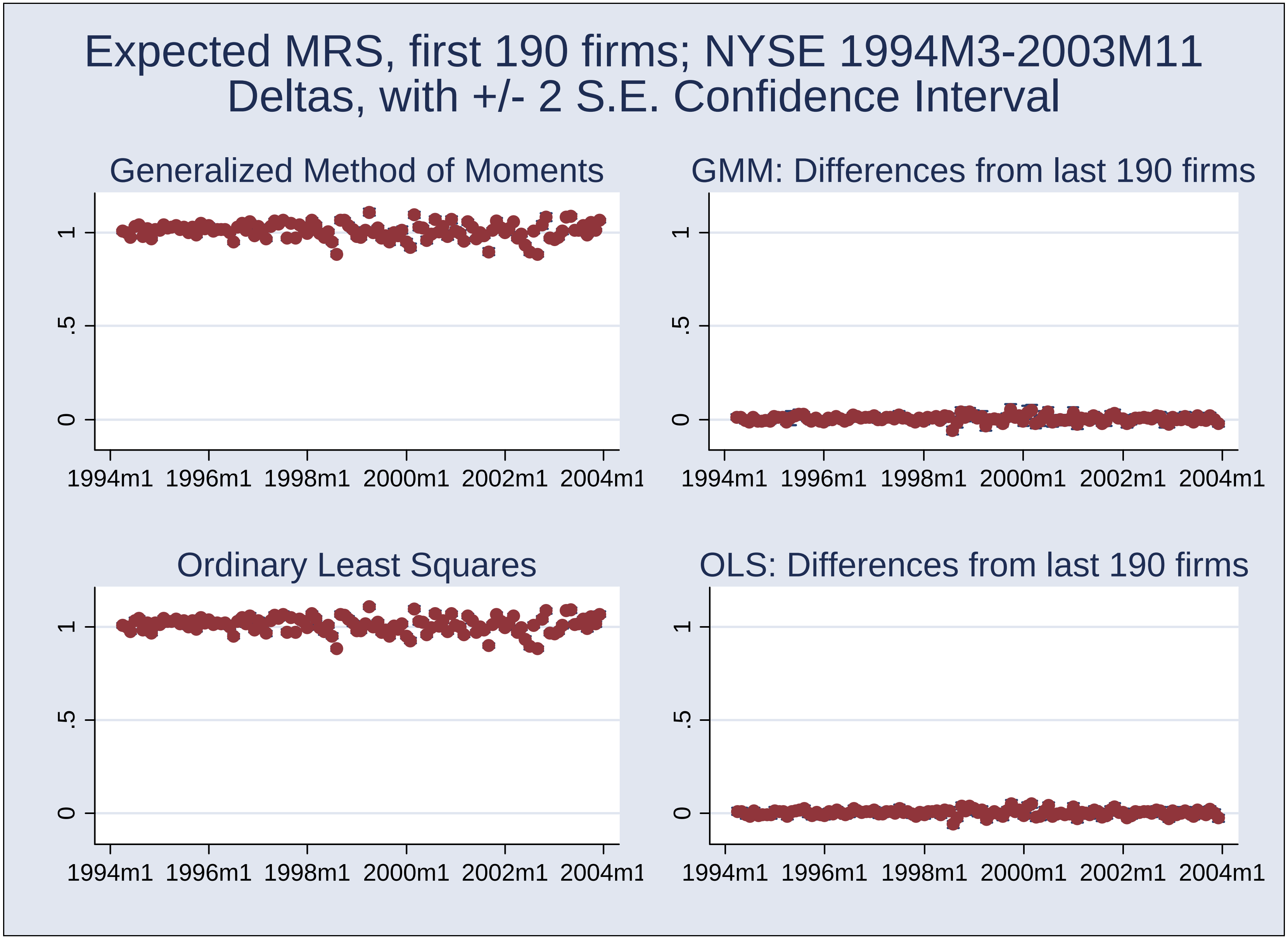

Figure 1: Estimated Expected MRS, Portfolios of S&P500 firms, 1994M3-2003M11: Different Estimators

Internal Integration

- Inside S&P 500, estimates of $\{\hat{\delta}\}$ from different sets of firms similar
- Can test for joint equivalence with F-test
    - Bootstrap because of non-normality (leptokurtosis)
    - Cannot reject equality within S&P 500 portfolios, any reasonable significance level
        - That is, do not reject integration

Comparison with T-bill

- Similar means
- T-bills are *much* less volatile than EMRS
- Easily reject equality of EMRS and T-bill-equivalent
    - F-test over 50!

Other Markets

- 150 firms from TSE
- Again, reasonable means, tight precision, much volatility
- Different estimators => similar results

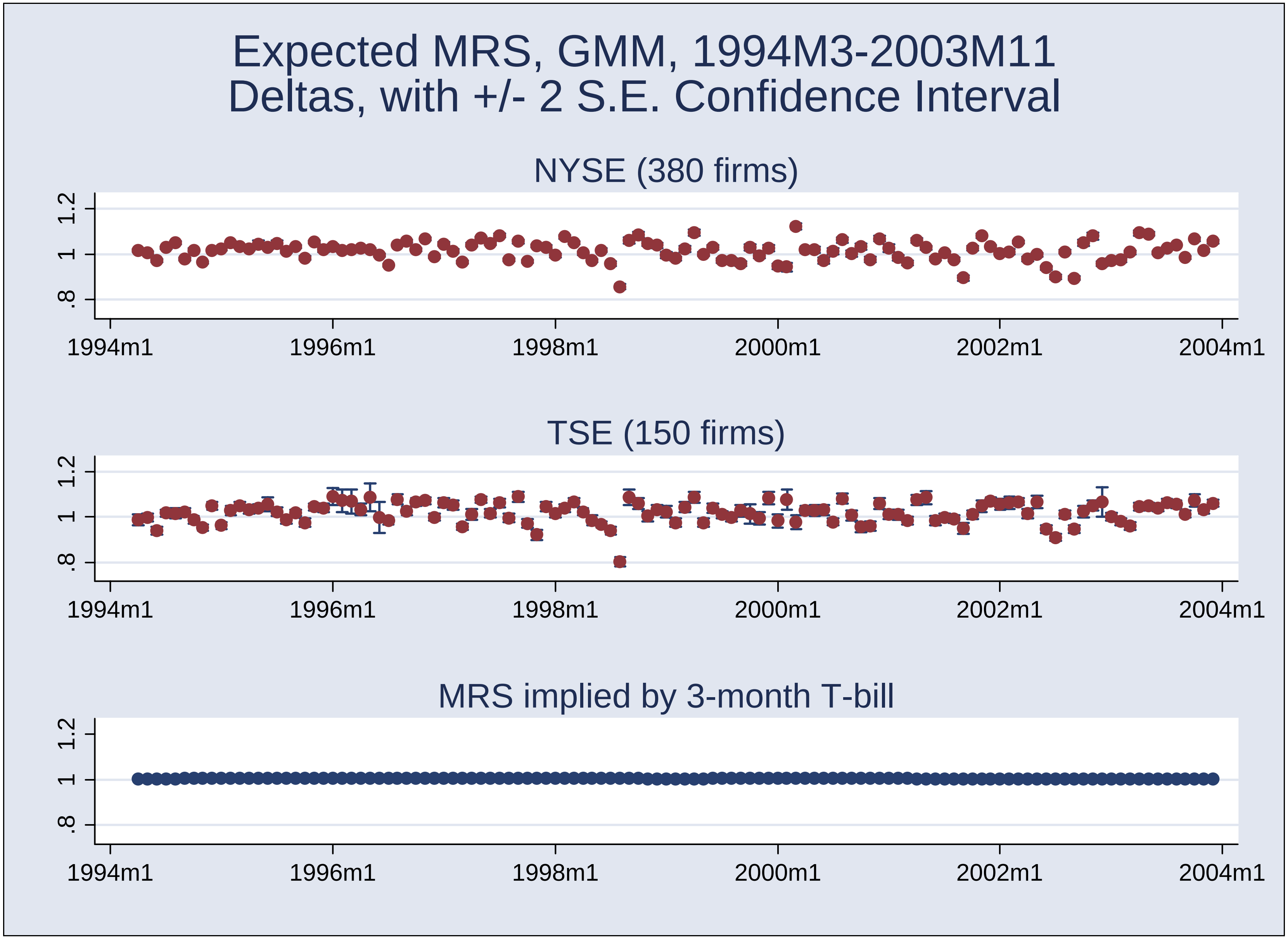

Figure 2: Estimates of Expected Marginal Rate of Substitution, 1994M3-2003M11: Different Markets

Integration: Comparing EMRS Across Different Markets

- Estimate EMRS from NYSE and TSE
- Estimates of EMRS are positively correlated across markets
  - Correlation of NYSE and TSE = .64
  - But mean absolute error = .03; 20% > .05
  - Can easily reject integration across markets
  - F-tests > 10, strong rejection

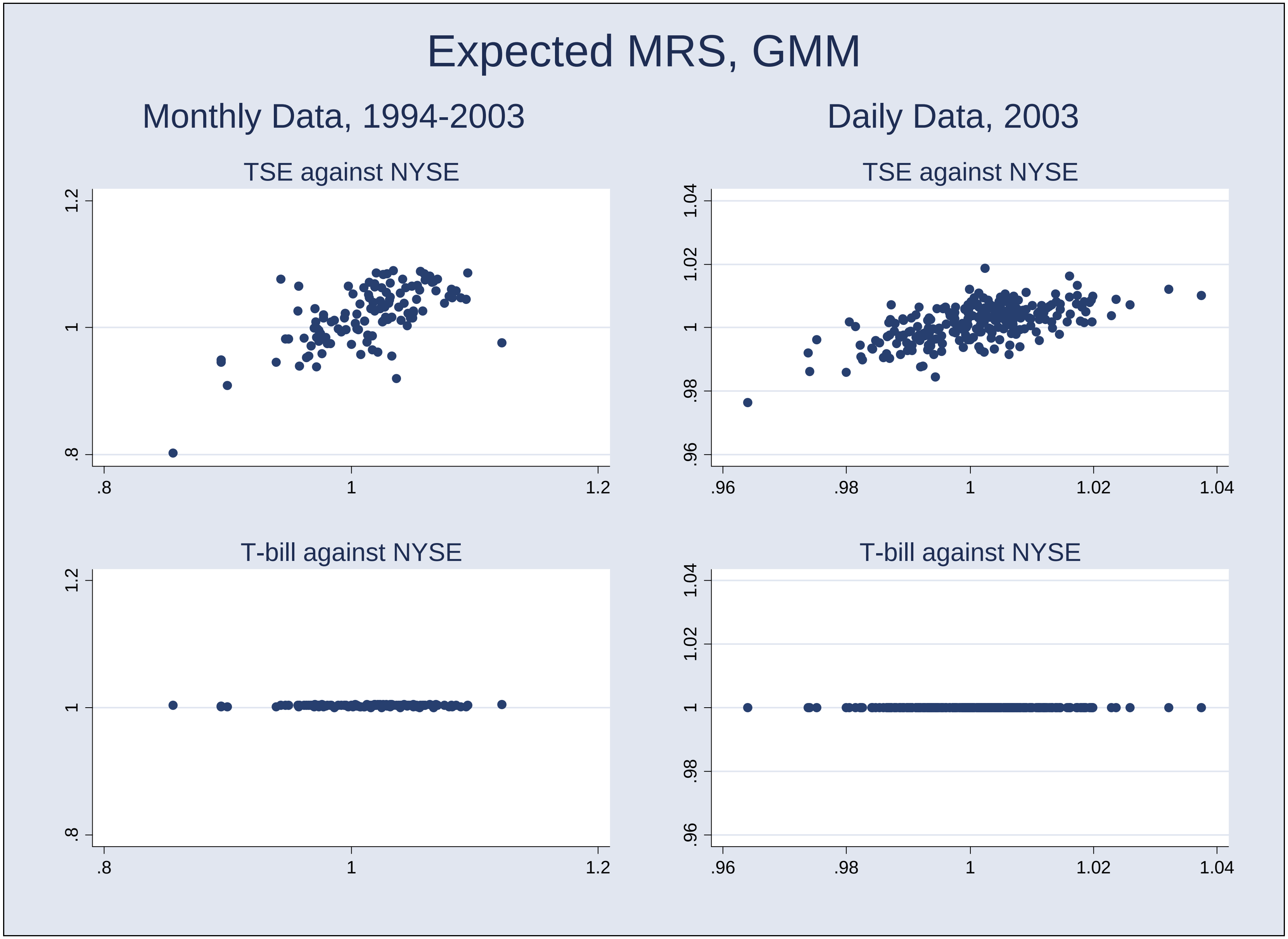

Figure 3: Scatter-plots of Estimated Expected MRS across Markets

Daily Results

- Similar to monthly results
- Reasonable EMRS, precisely estimated, great volatility
- Internal integration, but easily reject integration across markets
- Strongly reject equality with T-bills (too smooth!); F-test > 150
- EMRS positively correlated across markets
  - Still, easily reject integration across markets
  - F-tests integration of NYSE w/TSE > 20

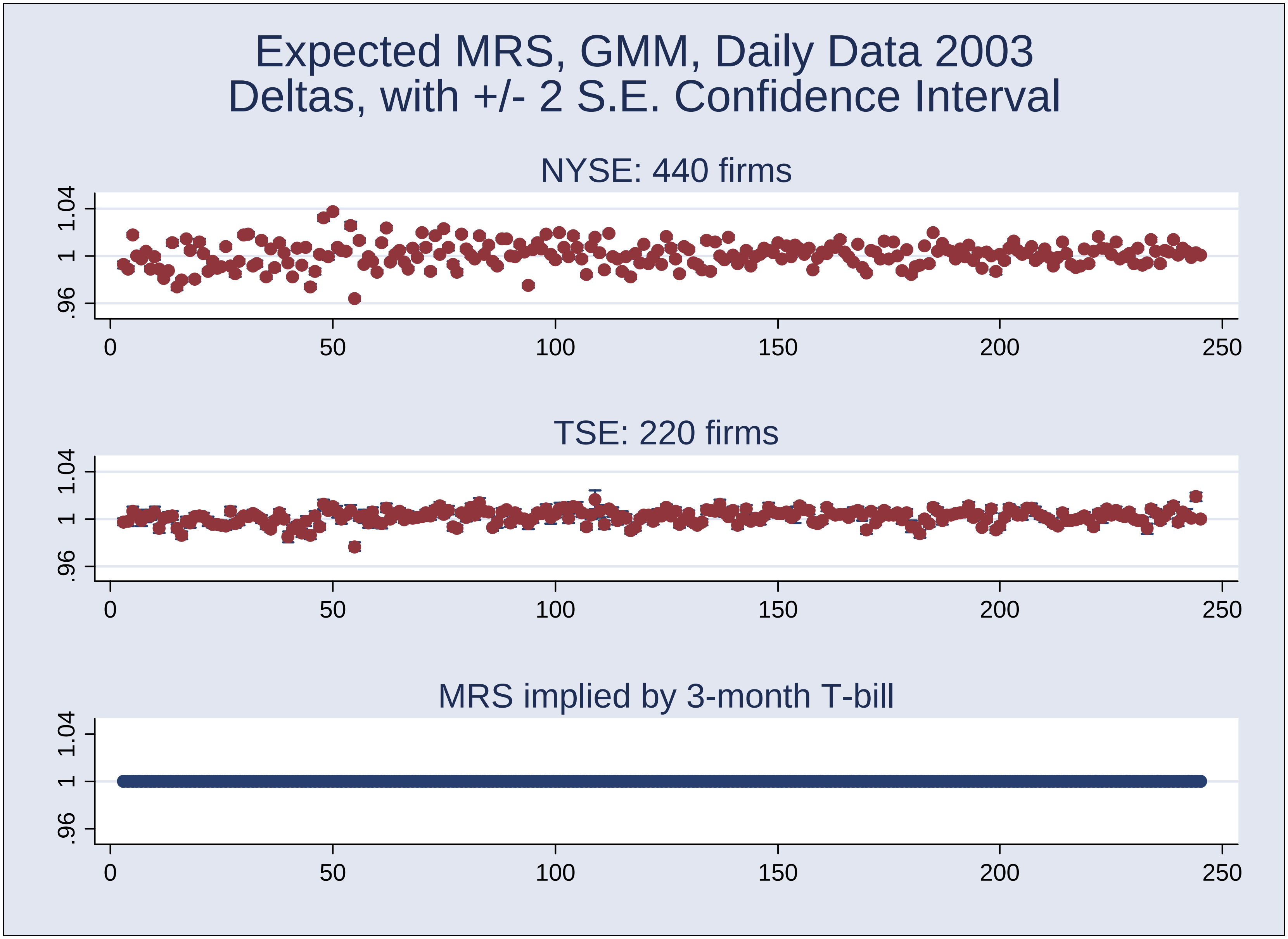

Figure 4: Daily Estimates of Expected Marginal Rate of Substitution, 2003

Future Agenda

- Adding Covariance Model?
- Different portfolio structure?
- Different model to estimate idiosyncratic risk?
  - Different normalization?
- Forward-looking test for arbitrage profits from diverging EMRS's across markets, lack of equality with t-bill
- Explain *reasons* for lack of integration